A Note about variable substitution

In each of the formulas I have gone over, each variable is exactly that, a variable. A variable is something that you can substitute in any value of the similar type for...that is what makes formulas with variables so powerful – they are applicable to a large number of cases. Here is an example of what I mean:

The Distributive Law is:

$\mathbf{p \lor (q \land r) \Leftrightarrow (p \lor q) \land (p \lor r)}$

Imagine simplifying the following expression:

$((s \land t) \lor v) \land (\neg v \lor (s \land t))$

$((s \land t) \lor v) \land ((s \land t) \lor \neg v)$, **Commutative**

$(s \land t) \lor (v \land \neg v)$, **Distributive, with** $p= s \land t$, $q= v$, $r=\neg v$

$(s \land t) \lor F$, **Inverse Law**

$(s \land t)$, **Identity Law**

One other thing to mention: notice how I used different letters in the problem than are listed in the rules...The reason I did that was for clarity's sake; to show you all what was substituted for each variable. BUT, in the problems that YOU do, you'll find that ps, qs and rs are used, just like in the formula. When you make substitutions, you'll have to differentiate between the p&qs from the formula, and the p&qs in your specific problem!!! Be careful when doing this.

Rules of Inference

1) p This is the Rule of Detachment or Modus Ponens

$p \Rightarrow q$

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q

This can be proved by the truth table below. Very simply, given a premise and an implication using that premise, the conclusion of the implication must follow.

p	q	$p \Rightarrow q$	$[p \wedge (p \Rightarrow q)] \Rightarrow q$
0	0	1	1
0	1	1	1
1	0	0	1
1	1	1	1

2) $p \Rightarrow q$ This is the Law of Syllogism

$q \Rightarrow r$

$p \Rightarrow r$

Consider the logical form of this law:

$$[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$$

Essentially, this reduces to showing
$$\neg[(p \Rightarrow q) \wedge (q \Rightarrow r)] \vee (p \Rightarrow r)$$

If p is false, we see the statement is true automatically. Thus we must only worry about the case where p is true. In this case, if r is true, we see we are fine as well. Thus, the final case to consider is if p is true and r is false. Regardless of q's value at least one of $(p \Rightarrow q)$ and $(q \Rightarrow r)$ must be false in this situation, making the entire assumption true.

3) $p \Rightarrow q$ **This is Modus Tollens**

$\neg q$

$\neg p$

In logical form we have $[(p \Rightarrow q) \wedge \neg q] \Rightarrow \neg p$

Here is a truth table to verify this rule:

p	q	$\neg p$	$\neg q$	$p \Rightarrow q$	$[(p \Rightarrow q) \wedge \neg q] \Rightarrow \neg p$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	0	1
1	1	0	0	1	1

The rest of the rules of implication are listed on page 79 of the book. You will get more practice using these in recitation.

Practice Problem :

Using the rules of inference, and given the following premises:

$p \Rightarrow (q \Rightarrow r)$

$p \vee s$

$t \Rightarrow q$

$\neg s$

Show that $\neg r \Rightarrow \neg t$ **must be true.**

Here is a list of the other rules stated in the text, without proof:

p
q
----
p ∧ q (Rule of Conjuction)

p ∨ q
¬p
----
q (Rule of Disjunctive Syllogism)

¬p → F
----
p (Rule of Contradiction)

p ∧ q
----
p (Rule of Conjunctive Simplification)

p
----
p ∨ q (Rule of Disjunctive Amplification)

p ∧ q
p → (q → r)
----
r (Rule of Conditional Proof)

$p \to r$
$q \to r$

$(p \lor q) \to r$ (Rule for Proof by Cases)

$p \to q$
$r \to s$
$p \lor r$

$q \lor s$ (Rule of the Constructive Dilemma)

$p \to q$
$r \to s$
$\neg q \lor \neg s$

$\neg p \lor \neg r$

Examples using Rules of Inference

Given the premises:

p
$p \rightarrow q$
$s \vee r$
$r \rightarrow \neg q$

Arrive at the conclusion:

$s \vee t$

Step	**Reason**
1) p	**Premise**
2) $p \rightarrow q$	**Premise**
3) q	**Modus Ponens (using 1, 2)**
4) $r \rightarrow \neg q$	**Premise**
5) $q \rightarrow \neg r$	**Contrapositive statement of 4**
6) $\neg r$	**Modus Ponenes (using 3, 5)**
7) $s \vee r$	**Premise**
8) s	**Disjunctive Syllogism (using 6, 7)**
9) $s \vee t$	**Disjunctive Amplification (using 8)**

If you'd like you can simply state all of your premises in the beginning and go from there.

Write the following arguments in symbolic form. (Assign p, q, r, etc. to simple statements and then translate the argument into multiple complex statements using p, q and r...) Then establish the validity of the argument or give a counterexample to show that it is invalid.

If I get my Christmas bonus AND my friends are free, I will take a road trip with my friends.. If my friends don't find a job after Christmas, then they will be free. I got my Christmas bonus and my friends did NOT find a job after Christmas. Therefore, I will take a road trip with my friends!

Assign the statements as follows:

q = "My friends are free."
p = "I get my Christmas bonus."
r = "I will take a road trip with my friends."
s = "My friends find a job after Christmas."

The argument, symbolically is as follows:

Premises:
$(p \wedge q) \rightarrow r$
$\neg s \rightarrow q$
p
$\neg s$

Conclusion: r

Steps	**Reasons**
1) $\neg s \rightarrow q$	**Premise**
2) $\neg s$	**Premise**
3) q	**Modus Ponens (using 1, 2)**
4) p	**Premise**
5) $p \wedge q$	**Conjunction**
6) $(p \wedge q) \rightarrow r$	**Premise**
7) r	**Modus Ponens (using 5, 6)**

Now, let's look at one more...

If I get my paycheck AND my mom lends me the car today, then I will go shopping (today). If I get to the bank before 4:00pm and it is a Friday, I get my paycheck. If my mom has the day off of work, she will lend me the car. Friday is my mom's day off work and it just so happens that today is Friday. Therefore, I will go shopping today.

Assign the statements as follows:

p = "I got my paycheck."
q = "My mom lends me the car."
r = "I will go shopping."
s = "I get to the bank before 4 pm."
t = "It is Friday."
u = "My mom has the day off work."

Symbolically, we have the premises:

$$p \wedge q \Rightarrow r$$
$$s \wedge t \Rightarrow p$$
$$u \Rightarrow q$$
$$t \Rightarrow u$$
$$t$$

And the conclusion:

∴ r

This is false however. Attempting to prove it will never lead to the conclusion r. Here is a truth assignment that makes all the premises true, but the conclusion false:

p = F
q = T
r = F
s = F
t = T
u = T

This corresponds to the situation where you do not get your paycheck, but your mother does lend you the car. You do not go shopping and do not get to the bank before 4 pm. It is Friday and your mother does have off work. Basically, none of the premises specifies that you must get to the bank before 4. If this doesn't occur, then you can not guarantee going shopping. (The guarantee would only occur if you did get to the bank before 4pm AND got your mother's car.)