

## **COT 3100 Final Exam Course Outline/Final Exam Topic Outline**

### **I. Boolean Logic**

- a. Logical Operators and Symbols**
- b. Truth Tables**
- c. Laws of Logic**
- d. Proving Laws of Logic via Truth Tables**
- e. Proving Logical Equivalence via Laws of Logic**
- f. Rules of Implication**
- g. Proving Arguments by Rules of Implication & Laws of Logic**
- h. Contrapositive, Inverse and Converse of a Statement**
- i. Quantifier meanings**
- j. Two Nested Quantifiers (four possible orderings)**

### **II. Sets**

- a. Symbols and Operators**
- b. Definitions based on logic operators**
- c. Use of Set Laws to Prove equivalence of sets**
- d. Use of Set Table to Prove equivalence of sets**
- e. Direct Proof of if-then propositions**
- f. Proof of Contrapositive of if-then propositions**
- g. Proof by Contradiction of if-then propositions**
- h. Use of Counter-Example for disproof.**
- i. Inclusion-Exclusion Principle for 2 and 3 sets**

### **III. Number Theory**

- a. Definition of Division**
- b. Use of division rules to prove/disprove divisibility**
- c. Use of division for base conversion**
- d. Definition of mod via division**
- e. Mod rules**
- f. Fast Modular Exponentiation by hand (and cycle method)**
- g. Linear Equation Solver + EEA + Modular Inverse**
- h. Fundamental Theorem of Arithmetic**
- i. Pi Notation**
- j. Proof  $\sqrt{2}$  is irrational**
- k. Proof of an infinite number of primes**
- l. GCD/LCM connection via prime factorization.**
- m. Number of Divisors of an integer via prime factorization.**
- n. Sum of Divisors of an integer via prime factorization.**
- o. Parity of number of divisors of an integer**
- p. Primality test based on max value of smallest non-trivial div**
- q. Prime Sieve**
- r. Number of times a prime divides evenly into  $n!$**

## **IV. Mathematical Induction and Prelims**

### **a. Summations**

- i. Constant**
- ii. Sum of  $i, i^2, i^3$**
- iii. Split sum idea**
- iv. Index shift idea**
- iii. Geometric sum**
- iv. Mixed/Hybrid sum via eqn mult and sub**

### **b. Matrices**

- i. How to add**
- ii. How to multiply**
- iii. How to exponentiate**

### **c. Recursively Defined Sequences**

- i. How to apply recursive definition**
- ii. Knowing difference between definition of a sequence and bounds of problems that contain a sequence.**

### **d. Principle of Mathematical Induction**

- i. Base Case**
- ii. Inductive Hypothesis**
- iii. Inductive Step**

### **e. Problem types**

- i. Summations, equality**
- ii. Summations, inequality**
- iii. Divisibility**
- iv. Matrices**
- v. Recurrence Relations**
- vi. Strong Induction**
- vii. Constructive**
- viii. Potpourri**

## **V. Counting**

- a. Addition**
- b. Subtraction**
- c. Multiplication**
- d. Division**
- e. Permutations (distinct and with repeats)**
- f. Permutation of k objects out of n distinct objects**
- g. Combinations**
- h. Perm/Combo in Problems**
  - i. Grid Walking**
  - ii. Separator idea**
  - iii. Super Letter idea**
  - iv. Don't forget subtraction!**
  - v. Combinatorial Proof**
  - vi. Pascal's Triangle**
  - vii. Binomial Theorem**
- i. Combinations with Repetition**
  - i. Regular Version**
  - ii. Restriction  $x \leq c$**
  - iii. Restriction  $x \geq c$**
  - iv. Two restrictions of the form  $x \geq c$**
  - v. Both (ii) and (iv) together.**
  - vi. Solving # of solutions with  $\leq$ . (slack variable)**

## **VI. Probability**

- a. Two Counting Problems**
- b. Sample Space – each item equally likely**
- c. Two Standard Dice Distribution**
- d. Notation**
- e. Conditional Probability Definition**
- f. Inclusion-Exclusion Principle**
- g. Mutually Exclusive Events**
- h. Independent Events**
- i. Bayes Law**
- j. Probability Trees for Bayes Law Problems**
- k. Sum of Probabilities Must Equal 1.**
- l. Let's Make a Deal**
- m. How Low can you Go**
- n. Use of “recursive interpretation” to solve first to hit target**
- o. Craps winning probability calculation**
- p. Discrete Random Variables**
  - i. Definition of Expectation**
  - ii. Definition of Variance, Standard Deviation**
  - iii. Two Formulas for Variance**
  - iv. Median, Mode**
- q. Continuous Random Variables**
  - i. Definition of Expectation**
  - ii. Definition of Variance, Standard Deviation**
  - iii. Two Formulas for Variance**
  - iv. Median, Mode**
- r. Use of Parity in Problems**
- s. Carefully listing out Cartesian Product Sample Space**
- t. Simplifying (s) based on parity**

## **VII. Relations**

- a. Definition**
- b. Properties + Proving them**
  - i. Reflexive**
  - ii. Irreflexive**
  - iii. Symmetric**
  - iv. Anti-symmetric**
  - v. Transitive**
- c. Equivalence Relations + Equivalence Classes**
- d. Partial Ordering Relations**
- e. Relation Composition**
- f. Proofs Dealing with Relation Composition**
- g. Reflexive, Symmetric, Transitive Closure**

## **VIII. Functions**

- a. Definition as a special type of Relations**
- b. Domain, Co-Domain, Range**
- c. Function Composition**
- d. Properties – Injection, Surjection, Bijection**
- e. Inverse Definition (only on bijections)**
- f. Inverse Computation**
- g. Inverse of a function composition**
- h. Roots/Coefficient Relationship upto cubic**
- i. Plug in  $x = 0$ ,  $x = 1$ ,  $x = -1$  to get info about coefficients**
- j. Remainder Theorem**

## **IX. Recitation Material**

- a. Distance = Rate x Time**
- b. Log Rules**
- c. Random Algebra**
- d. Roots of Polynomials connection to coefficients**
- e. Prime Factorization use in Problems**
- f. Arithmetic/Geometric Series**
- g. Counting (more techniques from problems)**
- h. Probability (more techniques from problems)**