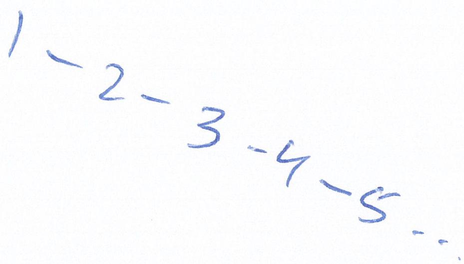
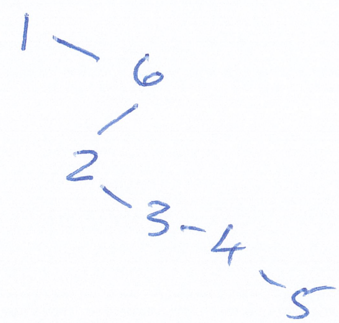
COP 3502 3/31/2026

AVL Trees

Issue w/ regular BST is search, insert, delete run in $O(h)$ time, h = height, WORST CASE $h = n-1$, n = # Nodes





2^{n-1} BSTs of n ints of height $n-1$.

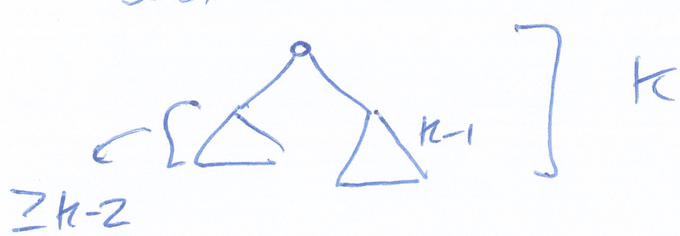
WORST CASE RUN-TIME IS $O(n)$ ops

AVG CASE IS $O(\lg n)$

Goal = guarantee $O(\lg n)$ height worst case

AVL Tree Node Property

1) regular BST (left lower, right higher)
2) for every node, the height of the left subtree + height of right subtree can't differ by more than 1.



Let T_h = min number of nodes for an AVL tree of height h.

Prove $T_h = F_{h+3} - 1$, where F_n is the n^{th} Fibonacci #.

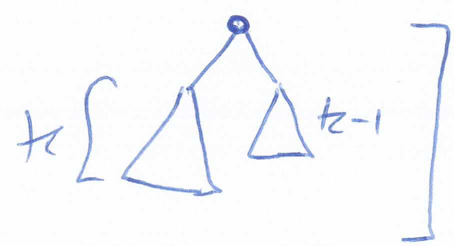
$h = 0$ $\circ$, $T_0 = 1$ $LHS = T_0 = 1$, $RHS = F_{0+3} - 1 = F_3 - 1 = 2 - 1 = 1$ ✓

$h = 1$, $T_1 = 2$ $LHS = T_1 = 2$, $RHS = F_{1+3} - 1 = F_4 - 1 = 3 - 1 = 2$ ✓

I.H. Assume for all non-neg int n, $n \leq k$ that $T_n = F_{n+3} - 1$ where k is an arbitrarily chosen positive integer.

I.S. Prove $\cancel{T_{n+1}} = T_{k+1} = F_{k+4} - 1$.

$$T_{k+1} = T_k + T_{k-1} + \underbrace{1}_{\text{root}}$$



$$= F_{k+3} - 1 + F_{k+2} - 1 + 1, \text{ using I.H.}$$

$$= F_{k+4} - 1 \checkmark$$

n = # nodes, h = height

$$n \geq F_{h+3} - 1 \qquad F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$$

$$n + 1 \geq F_{h+3} \qquad F_n \sim c\phi^n$$

$$n + 1 \geq c \cdot \phi^{h+3}$$

$$\phi^{h+3} \leq \frac{n+1}{c} \rightarrow h + 3 \leq \log_\phi \frac{n+1}{c}$$

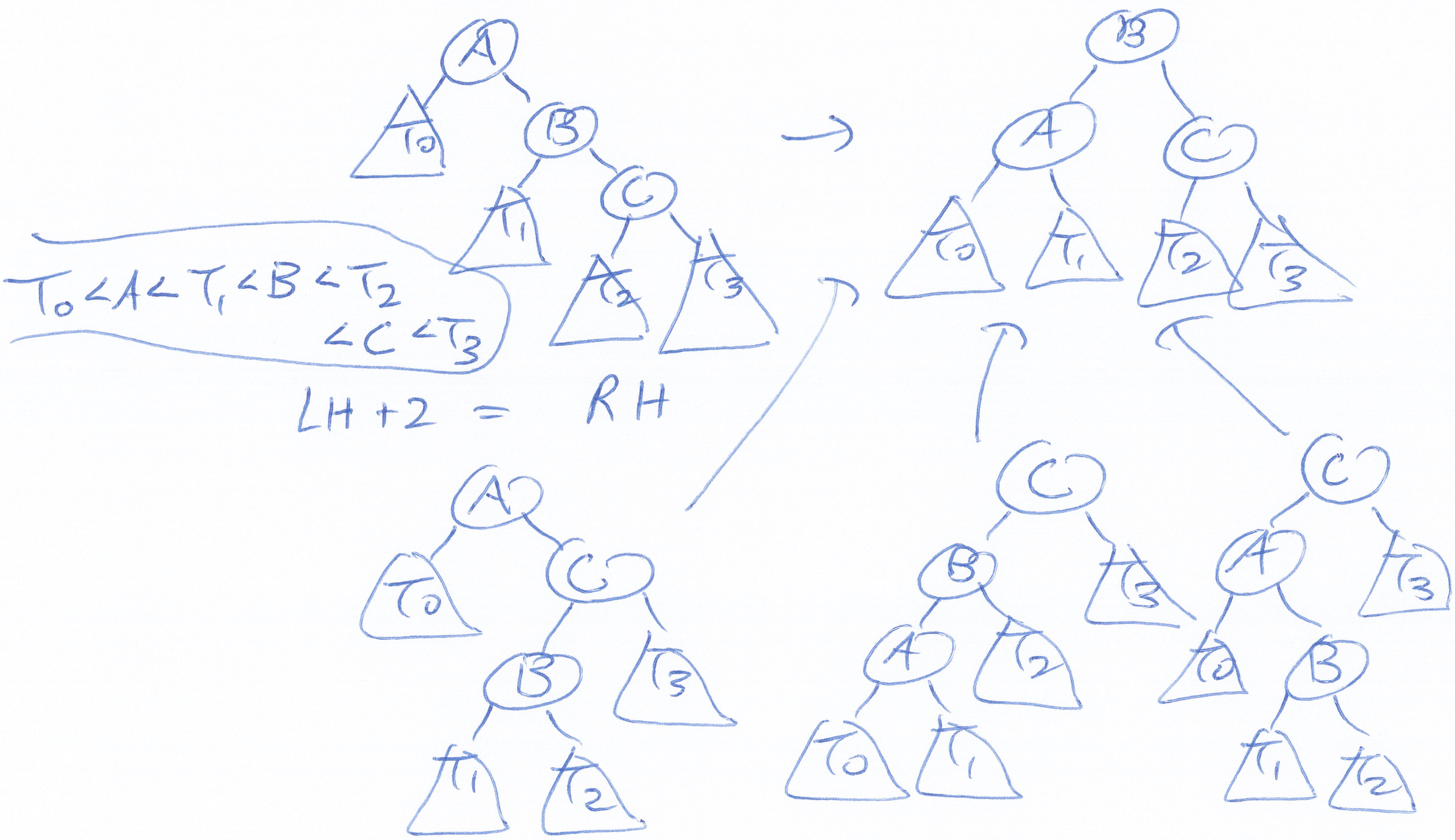
$$h + 3 \leq O(\lg n)$$

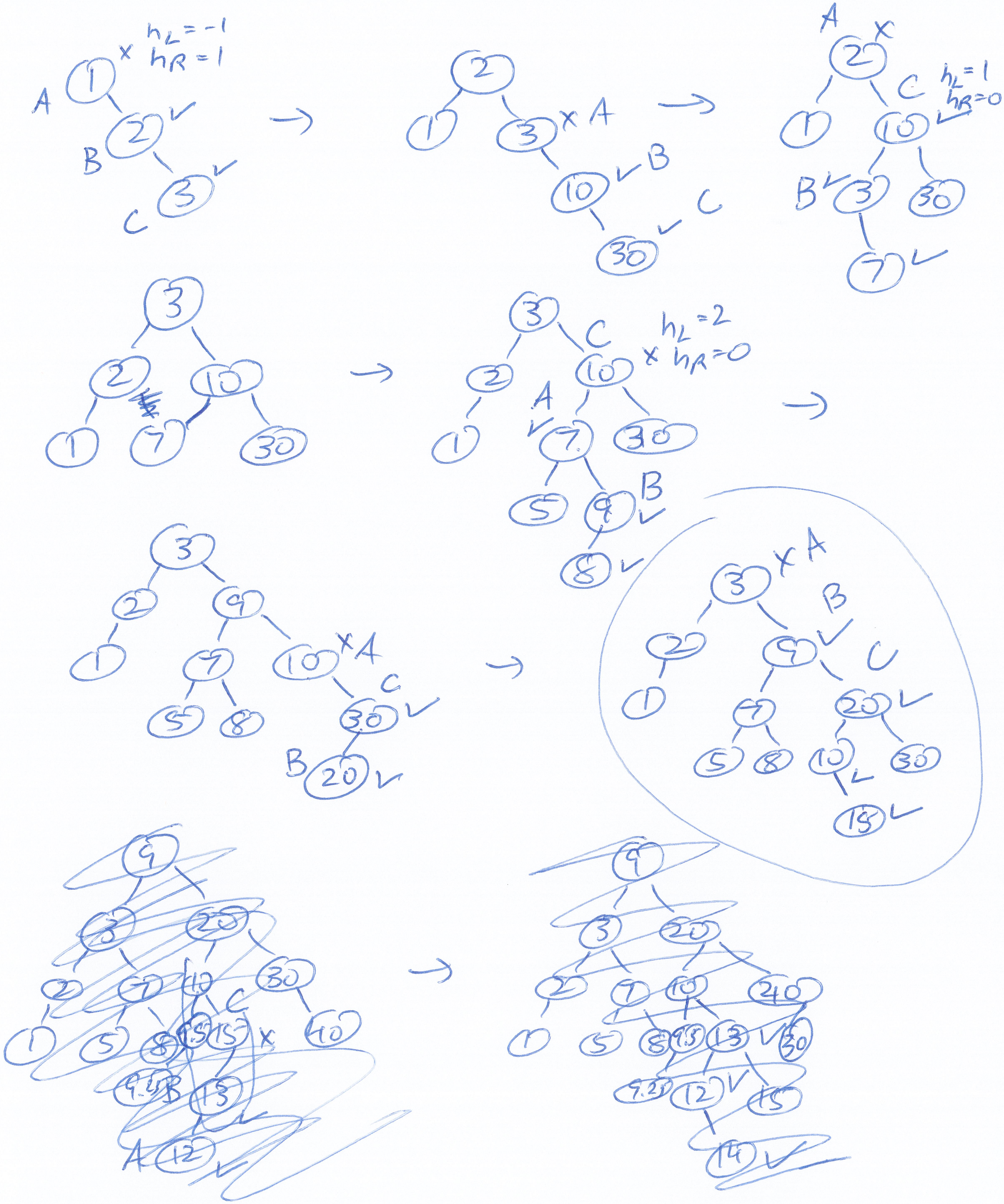
AVL

Insert - Step 1 - regular bst insert

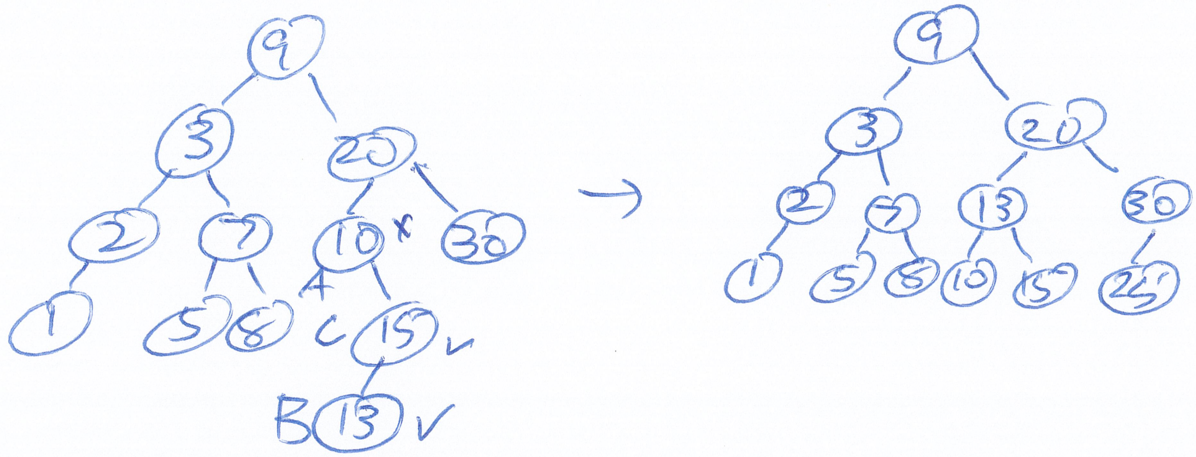
Step 2 - trace up the ancestral path and if there's an imbalance, fix it!

General Procedure to Fix





hL = -1
hR = 1
hL = 1
hR = 0
hL = 2
hR = 0



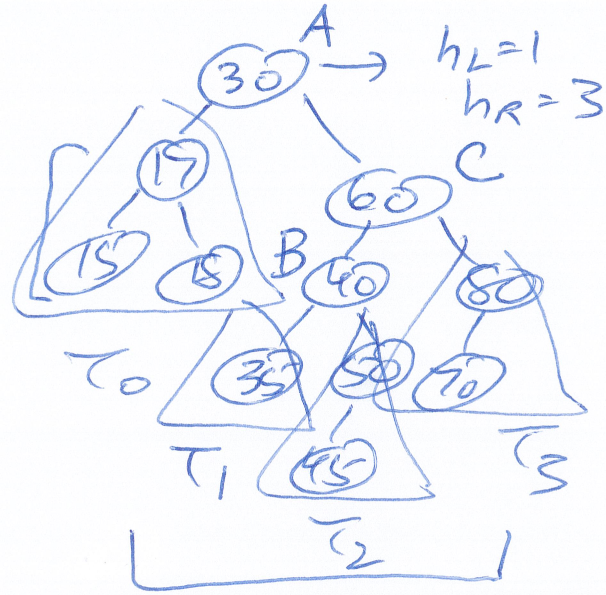
Delete

1) Do a regular BST Delete

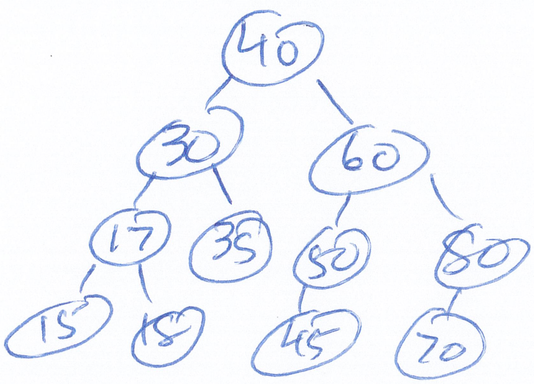
2) Trace up ancestral path of ~~paren~~ parent of deleted node and rebalance whenever there's a problem

→ Can trigger at each node it traces up.

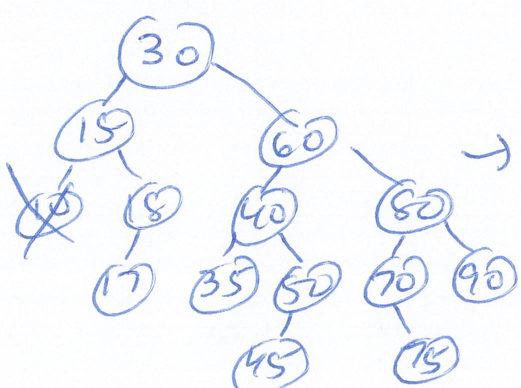
Delete 10



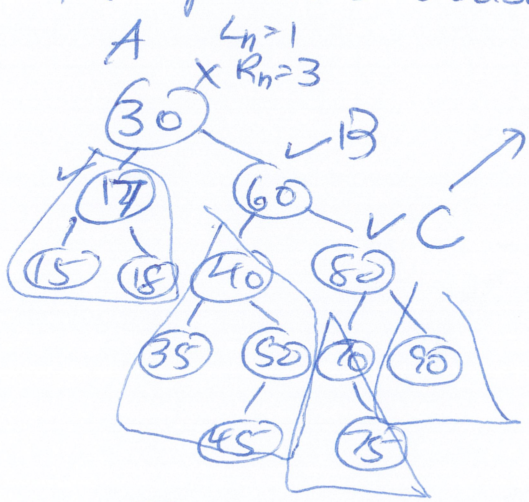
we choose C, B due to greater height



Note: I was right: when choosing A, B, C for delete you go to the longer subtree. If the 2 sides are equal, you ALWAYS go in the same direction as the previous decision. Here is a case

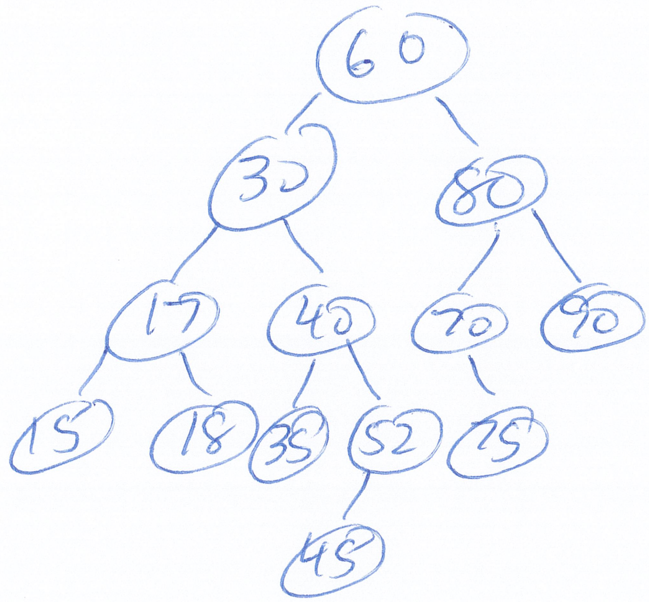


Del 10



we choose this because 60 is right of 30, so we went Right again in the tie case

Final answer on next page


60
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75
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