COP 3502 3/3/2026

$O(n^2)$ Sorts	Best	Avg	Worst
Bubble Sort	$O(n^2)$	$O(n^2)$	$O(n^2)$
Insertion Sort	$O(n)$	$O(n^2)$	$O(n^2)$
Selection Sort			$O(n^2)$

Bubble Sort

8, 2, 3, 9, 1, 5, 4, 7

2 8 3 9 1 5 4 7

2 3 8 9 1 5 4 7

2 3 8 9 1 5 4 7

2 3 8 1 9 5 4 7

2 3 8 1 5 9 4 7

2 3 8 1 5 4 9 7

2 3 8 1 5 4 7 [9]

1 iteration of Bubble Sort

Sort rest

Recursive

```
bubbleSort(arr, n) {
    OneRoundJoust(arr, n);
    bubbleSort(arr, n-1);
}
```

	8	2	3	9	1	5	4	7
Line 1	2	3	8	1	5	4	7	9
Line 2	2	3	1	5	4	7	8	9
Line 3	2	1	3	4	5	7	8	9
Line 4	1	2	3	4	5	7	8	9] SORTED
Line 5	1	2	3	4	5	7	8	9
Line 6	1	2	3	4	5	7	8	9
Line 7	1	2	3	4	5	7	8	9

Analysis: ~~#~~ + $(n-1) + (n-2) + \cdots + 1 = \frac{(n-1)n}{2} = O(n^2)$

# of Joust Iteration	# Josts Iteration	# Josts Iteration
1	2	$n-1$

Insertion Sort

Outer loop →

Inner ←

[6], 3, 7, 1, 5, 2, 4, 8	
[3 6] 7, 1, 5, 2, 4, 8	Iter 1
[3 6 7] 1, 5, 2, 4, 8	Iter 2
[1 3 6 7] 5, 2, 4, 8	Iter 3
[1 3 5 6 7] 2, 4, 8	Iter 4
[1 2 3 5 6 7] 4 8	Iter 5
[1 2 3 4 5 6 7] 8	Iter 6
[1 2 3 4 5 6 7 8]	Iter 7

Insertion Sort

worst = $1 + 2 + 3 + \cdots + (n-1) = \frac{(n-1)n}{2} = O(n^2)$

(1 = # swaps, (n-1) = # swaps)

best = $1 + 1 + 1 + \cdots + 1 = O(n)$

avg # swaps per iteration:

$$\frac{1}{2} + \frac{2}{2} + \frac{3}{2} + \cdots \frac{n-1}{2} = \frac{1}{2} \cdot \frac{(n-1)(n)}{2} = O(n^2)$$

Selection Sort

3	5	2	9	6	1	7		maxI = 0, +3
3	5	2	7	6	1	[9]	Iter 1	
3	5	2	1	6	[7 9]		Iter 2	max1 = 0, +3
3	5	2	1	[6 7 9]			Iter 3	
3	1	2	[5 6 7 9]				Iter 4	max1 = 0, +4
2	1	[3 5 6 7 9]					Iter 5	Outer →
[1 2 3 5 6 7 9]							Iter 6	Inner →

best/worst/avg = $n + (n-1) + \cdots + 1$

$= \frac{n(n+1)}{2} = O(n^2)$