COP 3502 1/29/26

Recursion Examples

- pow ✓
- tri ✓
- printchart ✓
- dicegame ✓
- towers ✓
- baseconv
- binsearch

pow

$b^e = b^{e-1} \times b$] pow(b,e) = pow(b, e-1) × b

e == 0 → ret 1

if $e < 0$, $b^e = \frac{1}{b^{-e}}$, $b^{-e} = \frac{1}{b^e}$

tri

$t(n) = 1 + 2 + 3 + \cdots (n-1) + n$

$t(n) = t(n-1) + n$, $t(0) = 0$

15.5

printchart (int start, int end, double ~~rate~~ perc)

10 20

	\$10	\$1.55	} print 1 line ①
start+1	\$11	\$1.71...	
end	\$20	\$3.10	

printchart ②

Dice Game Rec

```
int dicegamerolls(int prev, int maxside) {

    int roll = rand() % maxside + 1;
    if (roll < prev) {
        printf("you rolled %d, game over\n", roll);
        return 1;
    }

    return 1 + dicegamerolls(roll, maxside);

}
```

init call   dicegamerolls(0,6);

Analysis

Let $E(x)$ be avg # of turns

~~E(x~~. Define $E(x,i)$ = avg # turns given prev roll is i.

$$E(x) = 1 + \frac{1}{6}E(x,1) + \frac{1}{6}E(x,2) + \frac{1}{6}E(x,3) + \frac{1}{6}E(x,4) + \frac{1}{6}E(x,5) + \frac{1}{6}E(x,6)$$

(the 1 ↑ = 1st roll)

$$E(x,6) = 1 + \frac{1}{6}E(x,6)$$

$$E(x,6)\left(1 - \frac{1}{6}\right) = 1, \quad E(x,6) = \frac{1}{\frac{5}{6}} = \boxed{\frac{6}{5}}$$

Towers (n, s, e) → 1, 2 or 3 different

if (n <= 0) return

mid = 6 - s - e; //middle tower not s or e.

Towers (n-1, s, mid)

move disk n from tower s to tower e

Towers(n-1, mid, e)

Let T(n) = # moves for n disks

$T(1) = 1$

$T(n) = \underbrace{T(n-1)}_{\text{move } n-1} + \underbrace{1}_{\text{move bot}} + \underbrace{T(n-1)}_{\text{move } n-1}$

$T(n) = \boxed{2T(n-1) + 1}$

$= 2[2T(n-2) + 1] + 1$

$= 4T(n-2) + (2+1)$

$= \boxed{4T(n-2) + 3}$

$= 4(2T(n-3) + 1) + 3$

$= \boxed{8T(n-3) + 7}$

$T(n-1) = 2T(n-2) + 1$

$T(n-2) = 2T(n-3) + 1$

After k iterations we have $T(n) = 2^k T(n-k) + (2^k - 1)$

let $n - k = 1$, so $k = n - 1$ and substitute:

$T(n) = 2^{n-1} T(1) + (2^{n-1} - 1)$

$= 2^{n-1} + 2^{n-1} - 1 = \boxed{2^n - 1}$

$2^{n-1}(1+1) = 2^{n-1} \cdot 2^1 = 2^n$ works

Base Conversion

any base to base 10

binary base 2 (0,1) ternary base 3

octal base 8 (0,1,2,3,.. 7)

hexadecimal base 16 (0,1,.. 9,a,b,c,d,e,f)

$$\begin{array}{r} 64 \\ \times 7 \\ \hline 448 \\ 24 \\ 2 \\ \hline 474 \end{array}$$

$$732_8 = 7\times8^2 + 3\times8^1 + 2\times8^0 = 474_{10}$$

$$a_{k-1}a_{k-2}a_{k-3}\cdots a_1a_{0\,b} = \boxed{\sum_{i=0}^{k-1} a_i \times b^i}$$

Convert to base 10

$$00111011_2 = 0\times2^7 + 0\times2^6 + 1\times2^5 + 1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0$$

$$= 32+16+8+2+1$$

$$= 59_{10}$$

$$\text{convert}(a_{k-1}\cdots a_0, b) = a_0 + b * \text{convert}(a_{k-1}\cdots a_1, b)$$

a_0 ↑ ~~ls~~ least sig symbol

$b*$ → rest is shifted by 1 spot (mult b)

$$732_8 = 8\times73_8 + 2$$

Opposite process base 10 to base b

$$474_{10} = \underline{732}_8$$

$$474 = \boxed{a_2 8^2 + a_1 8} + a_0 \qquad (8\times(59) + 2)$$

↓ divisible by 8

so 474 % 8 must equal a_0

$$\begin{array}{l} 8\,\underline{|474} \\ 8\,\underline{|59} \quad R2 \uparrow \\ 8\,\underline{|7} \quad\; R3 \\ \quad 0 \quad\; R7 \end{array}$$

Normally base b_1 to b_2 — $b_1 \neq b_2$, $b_1 \neq 10$, $b_2 \neq 10$

$b_1 \to 10 \to b_2$

Exception is if b_1, b_2 are both powers of 2. (or 3 ...)

$32103_4 = 1110010011_2 = 1623_8$

$ca1f65_{16} = \_\_\_\__8 = \_\_\_\__4 = \_\_\_\__2$

$32103_4 = 3\times4^4 + 2\times4^3 + 1\times4^2 + 0\times4^1 + 3\times4^0$

$= (2^1+2^0)2^8 + (2^1)2^6 + (2^0)\times2^4 + (2^1+2^0)2^0$

$= \underbrace{2^9+2^8}_{3} + \underbrace{2^7}_{2} + \underbrace{2^4}_{1} + \underbrace{2^1+2^0}_{3}$ (binc

$= 1100\ 1010\ 0001\ 1111\ 0110\ 0101_2$

$= 6\ 2\ 4\ 1\ 7\ 5\ 4\ 5_8$