The Law of Signs

Stan’s Signs is a huge, nationwide supplier of signage for all kinds of businesses who want to advertise along the country’s highways. Stan makes signs in all kinds of shapes and sizes, but his best seller has always been his triangular signs. Stan believes that the key to his success is his quality control. You see, Stan insists that his triangle signs all conform to what he calls the “Law of Signs.” The Law of Signs simply states that the ratio of the length of one of the sides of the sign and the sine of the angle opposite that side must be the same for all three angle/side pairs. This may sound similar to another law that you’ve heard from analytic geometry, but Stan insists that it’s company proprietary. To better understand the Law of Signs, here’s a diagram:

\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

Stan realizes that his Law of Signs is ambiguous, in that two different angles can have the same sin value (45 degrees and 135 degrees, for example), so to account for this, he only sells acute triangular signs (all of the sign’s angles are less than 90 degrees).

Unfortunately, Stan’s order database recently suffered a hard drive crash, and the tape backup system failed to run that weekend, so Stan had to resort to a data recovery service to get his orders back. The service couldn’t restore the data completely, so all Stan has now is partial measurements for all the signs that must be made. He’s hired you to write a program to rebuild the database. Stan is clear that you’re only to use the Law of Signs and the angle sum property. You protest that other techniques would also be helpful in solving this problem, but Stan is worried about copyright infringement law suits from other sign companies. After a bit of convincing, Stan agrees to also let you use the triangle’s angle sum property, which states that all three angles of a triangle always sum to 180 degrees.

The Problem:

Given a partial set of angle and side measurements for each sign, reconstruct the complete measurements for the sign, such that they conform to the Law of Signs and the angle sum property. Some data may have been corrupted, so if the measurements can’t possibly be reconciled with each other (to within 0.01 inches), you must reject the order, and the customer must order again (a slightly annoyed customer is much better than an inferior Stan’s Signs product!) Also, in some cases there may not be enough recovered data to be able to reconstruct all the measurements using just the Law of Signs and the angle sum property. You’ll need to identify these cases as well.
The Input:

There will be multiple data sets. Each data set consists of one line containing six positive real numbers, \((A, B, C, a, b, c)\) separated by whitespace. The first three numbers represent the angle measurements of the sign. Angles will be given in degrees (greater than 0 and less than 90), and specified in counter-clockwise order. The second three numbers represent the lengths of the sides (given in inches) opposite those angles, respectively. If any of the measurements are negative, it means that that measurement is missing. Input is terminated by a line containing only six negative numbers.

The Output:

For each data set, print “Sign #n:”, where \(n\) is the number of the sign (starting with 1), followed by a space and the reconstructed set of three angle measurements and three side measurements, given in the same order as the input data. Print the measurements rounded to two decimal places (0.014 rounds to 0.01 and 0.015 rounds to 0.02). Separate each measurement by a single space. If a sign that conforms to both the Law of Signs and the angle sum property cannot be reconstructed, print “Rejected!” in place of the measurements. If there aren’t enough data to reconstruct all measurements using the techniques described, print “Lost!” in place of the measurements.

Sample Input:

\[
\begin{align*}
60.0 & \quad -1.0 & \quad -1.0 & \quad 36.0 & \quad -1.0 & \quad 36.0 \\
-1.0 & \quad 65.0 & \quad -1.0 & \quad 40.0 & \quad -1.0 & \quad 50.0 \\
75.0 & \quad 55.0 & \quad -1.0 & \quad 10.0 & \quad -1.0 & \quad 10.0 \\
-1.0 & \quad -1.0 & \quad -1.0 & \quad -1.0 & \quad -1.0 & \quad -1.0
\end{align*}
\]

Sample Output:

Sign #1: 60.00 60.00 60.00 36.00 36.00 36.00
Sign #2: Lost!
Sign #3: Rejected!