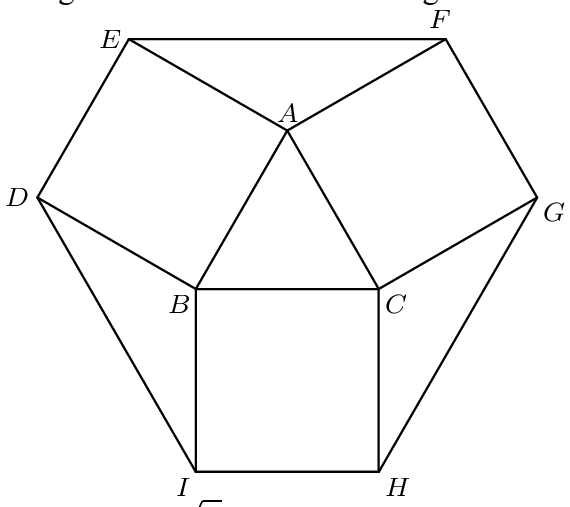
30-60-90 Triangles, Regular Triangle, Hexagon Areas

1) AMC 2014 10A - #13

Equilateral $\triangle ABC$ has side length 1, and squares $ABDE, BCHI, CAFG$ lie outside the triangle. What is the area of hexagon $DEFGHI$?



$\textbf{(A)}\ \frac{12+3\sqrt{3}}{4} \qquad \textbf{(B)}\ \frac{9}{2} \qquad \textbf{(C)}\ 3+\sqrt{3} \qquad \textbf{(D)}\ \frac{6+3\sqrt{3}}{2} \qquad \textbf{(E)}\ 6$

2) 2013 AMC 10B - #15

A wire is cut into two pieces, one of length a and the other of length b. The piece of length a is bent to form an equilateral triangle, and the piece of length b is bent to form a regular hexagon.

The triangle and the hexagon have equal area. What is $\frac{a}{b}$?

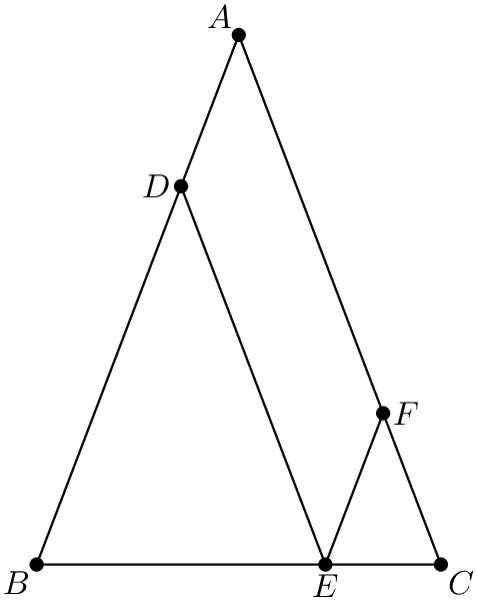
$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ \frac{\sqrt{6}}{2} \qquad \textbf{(C)}\ \sqrt{3} \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ \frac{3\sqrt{2}}{2}$

Reflection Trick

1) Mirror Cowboy

2) 2013 AMC 10 A - #12

In $\triangle ABC$, $AB = AC = 28$ and $BC = 20$. Points $D, E,$ and F are on sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, respectively, such that $\overline{DE}$ and $\overline{EF}$ are parallel to $\overline{AC}$ and $\overline{AB}$, respectively. What is the perimeter of parallelogram $ADEF$?



(A) 48 **(B)** 52 **(C)** 56 **(D)** 60 **(E)** 72

Use of Triangle Area

1) Which of the following (answer could be more than 1) could be the three altitudes of a non-degenerate triangle?

a) 2, 3, 6
b) 10, 11, 23
c) 5, 10, 10

2) 2013 AMC 10A - #15

Two sides of a triangle have lengths 10 and 15. The length of the altitude to the third side is the average of the lengths of the altitudes to the two given sides. How long is the third side?

(A) 6 **(B)** 8 **(C)** 9 **(D)** 12 **(E)** 18

Visualizing Rotations

1) 2013 AMC 10A - #20

A unit square is rotated 45° about its center. What is the area of the region swept out by the interior of the square?

$\textbf{(A)}\ 1-\frac{\sqrt{2}}{2}+\frac{\pi}{4}\qquad \textbf{(B)}\ \frac{1}{2}+\frac{\pi}{4}\qquad \textbf{(C)}\ 2-\sqrt{2}+\frac{\pi}{4}\qquad \textbf{(D)}\ \frac{\sqrt{2}}{2}+\frac{\pi}{4}\qquad \textbf{(E)}\ 1+\frac{\sqrt{2}}{4}+\frac{\pi}{8}$

Use of Pythagorean Theorem Twice

1) 2014 AMC 10B - #21

Trapezoid $ABCD$ has parallel sides $\overline{AB}$ of length 33 and $\overline{CD}$ of length 21. The other two sides are of lengths 10 and 14. The angles A and B are acute. What is the length of the shorter diagonal of $ABCD$?

$\textbf{(A)}\ 10\sqrt{6}\qquad \textbf{(B)}\ 25\qquad \textbf{(C)}\ 8\sqrt{10}\qquad \textbf{(D)}\ 18\sqrt{2}\qquad \textbf{(E)}\ 26$

Power of a Point

1) 2013 AMC10A - #23

In $\triangle ABC$, $AB = 86$, and $AC = 97$. A circle with center A and radius AB intersects $\overline{BC}$ at points B and X. Moreover $\overline{BX}$ and $\overline{CX}$ have integer lengths. What is BC?

$\textbf{(A)}\ 11\qquad \textbf{(B)}\ 28\qquad \textbf{(C)}\ 33\qquad \textbf{(D)}\ 61\qquad \textbf{(E)}\ 72$