**OMC Number Theory Homework Assigned on 10/14/2025**
**Due via email to dmarino@ucf.edu on 10/19/2025**

**1. Problem (2006 AMC 10B #22)**

Elmo makes $N$ sandwiches for a fundraiser. For each sandwich he uses $B$ globs of peanut butter at $4¢$ per glob and $J$ blobs of jam at $5¢$ per blob. The cost of the peanut butter and jam to make all the sandwiches is $\$2.53$. Assume that $B$, $J$, and $N$ are positive integers with $N > 1$. What is the cost of the jam Elmo uses to make the sandwiches?

$\text{(A) } \$1.05 \qquad \text{(B) } \$1.25 \qquad \text{(C) } \$1.45 \qquad \text{(D) } \$1.65 \qquad \text{(E) } \$1.85$

**2. Problem (2006 AMC 10B #25)**

Mr. Jones has eight children of different ages. On a family trip his oldest child, who is 9, spots a license plate with a 4-digit number in which each of two digits appears two times. "Look, daddy!" she exclaims. "That number is evenly divisible by the age of each of us kids!" "That's right," replies Mr. Jones, "and the last two digits just happen to be my age." Which of the following is ***not*** the age of one of Mr. Jones's children?

$\text{(A) } 4 \qquad \text{(B) } 5 \qquad \text{(C) } 6 \qquad \text{(D) } 7 \qquad \text{(E) } 8$

**3. Problem (2007 AMC10A #17)**

Suppose that $m$ and $n$ are positive integers such that $75m = n^3$. What is the minimum possible value of $m + n$?

$\text{(A) } 15 \qquad \text{(B) } 30 \qquad \text{(C) } 50 \qquad \text{(D) } 60 \qquad \text{(E) } 5700$

**4. Problem (2007 AMC10A #23)**

How many ordered pairs $(m, n)$ of positive integers, with $m \geq n$, have the property that their squares differ by $96$?

$\text{(A) } 3 \qquad \text{(B) } 4 \qquad \text{(C) } 6 \qquad \text{(D) } 9 \qquad \text{(E) } 12$

**5. Problem (2007 AMC 12A #11)**

A finite sequence of three-digit integers has the property that the tens and units digits of each term are, respectively, the hundreds and tens digits of the next term, and the tens and units digits of the last term are, respectively, the hundreds and tens digits of the first term. For example, such a sequence might begin with the terms 247, 475, and 756 and end with the term 824. Let $S$ be the sum of all the terms in the sequence. What is the largest prime factor that always divides $S$?

$\text{(A) } 3 \qquad \text{(B) } 7 \qquad \text{(C) } 13 \qquad \text{(D) } 37 \qquad \text{(E) } 43$

**6. Problem (2007 AMC 12A #12)**

Integers $a, b, c,$ and $d$, not necessarily distinct, are chosen independently and at random from 0 to 2007, inclusive. What is the probability that $ad - bc$ is even?

$\text{(A)}\ \frac{3}{8} \qquad \text{(B)}\ \frac{7}{16} \qquad \text{(C)}\ \frac{1}{2} \qquad \text{(D)}\ \frac{9}{16} \qquad \text{(E)}\ \frac{5}{8}$

**7. Problem (2007 AMC 12A #22)**

For each positive integer $n$, let $S(n)$ denote the sum of the digits of $n$. For how many values of $n$ is $n + S(n) + S(S(n)) = 2007?$

$\text{(A)}\ 1 \qquad \text{(B)}\ 2 \qquad \text{(C)}\ 3 \qquad \text{(D)}\ 4 \qquad \text{(E)}\ 5$

**8. Problem (1992 AHSME #17)**

The 2-digit integers from 19 to 92 are written consecutively to form the integer $N = 192021 \cdots 9192$. Suppose that $3^k$ is the highest power of 3 that is a factor of $N$. What is $k$?

$\text{(A)}\ 0 \quad \text{(B)}\ 1 \quad \text{(C)}\ 2 \quad \text{(D)}\ 3 \quad \text{(E)}\ \text{more than } 3$

**9. Problem (2000 AIME #1)**

Find the least positive integer $n$ such that no matter how $10^n$ is expressed as the product of any two positive integers, at least one of these two integers contains the digit $0$.

**10. Problem (2003 AMC 10A #25)**

Let $n$ be a $5$-digit number, and let $q$ and $r$ be the quotient and the remainder, respectively, when $n$ is divided by $100$. For how many values of $n$ is $q + r$ divisible by $11$?

$\text{(A)}\ 8180 \qquad \text{(B)}\ 8181 \qquad \text{(C)}\ 8182 \qquad \text{(D)}\ 9000 \qquad \text{(E)}\ 9090$