<u>Number Theory – AIME Preparation OMC 1/17/2025</u>

1. Review: GCD, LCM, Divisibility, # of Divisors, Sum of Divisors, Prime Fact of n!
2. Use of Mod
3. Fermat's Theorem
4. Euler Phi Function, Euler's Theorem

<u>Problems to go with Lecture Topics (104 Number Theory Problems)</u>

3) What is the largest positive integer n for which n^3+100 is divisible by n+10?

13) Compute the sum of all numbers of the form a/b, where a and b are relatively prime positive divisors of 27,000.

18) Find all positive integers n for which n! + 5 is a perfect cube.

<u>Problems to go with Lecture Topics (from Old AIMEs)</u>

1983-8) What's the largest 2 digit prime factor of $\binom{200}{100}$?

1989-9) One of Euler's conjectures was disproved in the 1960s by three American mathematicians when they showed that there is a positive integer n such that

$$133^5 + 110^5 + 84^5 + 27^5 = n^5$$

Find the value of n.

Problems from recent AIMEs

2020 AIME-I 10) Let m and n be positive integers satisfying the conditions

1. gcd(m+n, 210) = 1
2. m^m is a multiple of n^n, and
3. m is not a multiple of n,

Find the least possible value of m + n.

2020 AIME-II 10) Find the sum of all positive integers n such that when $\sum_{i=1}^{n} i^3$ is divided by n+5, the remainder is 17.

2021 AIME-II 9) Find the number of ordered pairs (m, n) such that m and n are positive integers in the set {1, 2, 3, …, 30} and the greatest common divisor of $2^m + 1$ and $2^n - 1$ is not 1.

2022 AIME-I 13) Let S be the set of all rational numbers that can be expressed as a repeating decimal in the form $0.\overline{abcd},$ where at least one of the digits $a, b, c,$ or d is nonzero. Let N be the number of distinct numerators obtained when numbers in S are written as fractions in lowest terms. For example, both 4 and 410 are counted among the distinct numerators for numbers in S because $0.\overline{3636} = \frac{4}{11}$ and $0.\overline{1230} = \frac{410}{3333}.$ Find the remainder when N is divided by $1000.$