Counting/Combinatorics – AIME Preparation OMC Homework for 1/3/2024

Problems from Old AIMEs

1997-2) The nine horizontal and nine vertical lines on a 8 x 8 checker board form *r* rectangles, of which *s* are squares. The number *r*/*s* can be written in the form *m*/*n*, where *m* and *n* are relatively prime positive integers. Find *m* + *n*.

1993-11) Alfred and Bonnie play a game in which they take turns tossing a fair coin. The winner of a game is the first person to obtain a head. Alfred and Bonnie play this game several times with the stipulation that the loser of a game goes first in the next game. Suppose that Alfred goes first in the first game, and that the probability that he wins the sixth game is m/n. where m and n are relatively prime positive integers. What are the last three digits of m + n?

1994-9) A solitaire game is played as follows. Six distinct pairs of matched tiles are placed in a bag. The player randomly draws tiles one at a time from the bag and retains them, except that matching tiles are put aside as soon as they appear in the player's hand. The game ends if the player ever holds three tiles, no two of which match; otherwise the drawing continues until the bag is empty. The probability that the bag will be emptied is p/q, where p and q are relatively prime positive integers. Find $p + q$.

1995-15) The *p* be the probability that, in the process of repeatedly flipping a fair coin, one will encounter a run of 5 heads before one encounters a run of 2 tails. Given that *p* can be written in the form *m*/*n*, where *m* and *n* are relatively prime positive integers, find *m* + *n*.

Problems from recent AIMEs

2021-I-1) Zou and Chou are practicing their 100 meter sprints by running 6 races against each other. Zou wins the first race, and after that, the probability that one of the wins a race is 2/3 is they won the previous race, but only 1/3 if they lost the previous race. The probability that Zou will win exactly 5 of 6 race six m/n, where m and n are relatively prime positive integers. Find m + n.

2022-I-9) Elina has twelve blocks, two each of red (R), blue (B), yellow (Y), green (G), orange (O), and purple (P). Call an arrangement of blocks ***even*** if there is an even number of blocks between each pair of blocks of the same color. For example, the arrangement

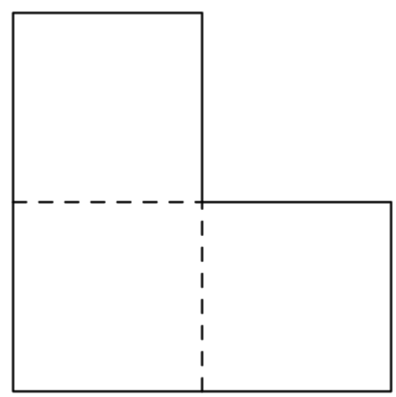
R B B Y G G Y R O P P O

Is even. Elina arranges her blocks in a row in random order. The probability her arrangement is even is m/n, where m and n are relatively prime positive integers. Find m + n.

2023-I-1) Five men and nine women stand equally spaced around a circle in random order. The probability that every man stands diametrically opposite a woman is m/n, where m and n are relatively prime positive integers. Find m + n.

2023-I-6) Alice knows that 3 red cards and 3 black cards will be revealed to her one at a time in random order. Before each card is revealed, Alice must guess its color. If Alice plays optimally, the expected number of cards she will guess correctly is m/n, where m and n are relatively prime positive integers. Find m + n.

2023-II-6) Consider the L-shaped region formed by three unit squares joined at their sides, as shown below. Two points A and B are chosen independently and uniformly at random from inside the region. The probability that the midpoint of $\overline{AB}$ also lies inside this L-shaped region can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.



2024-I-4) Jen enters a lottery by picking 4 distinct numbers from S={1,2,3,…,9,10}. Four numbers are randomly chosen from S. She wins a prize if at least two of her numbers were 2 of the randomly chosen numbers, and wins the grand prize if all four of her numbers were the randomly chosen numbers. The probability of her winning the grand prize given that she won a prize is $\tfrac{m}{n}$ where m and n are relatively prime positive integers. Find $m+n$.

2024-I-11) Each vertex of a regular octagon is independently colored either red or blue with equal probability. The probability that the octagon can then be rotated so that all of the blue vertices end up at positions where there were originally red vertices is $\tfrac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?