

Counting/Combinatorics – AIME Preparation OMC 1/3/2024

1. Combination Definition/Review – combo book 1, generalization
2. Inclusion/Exclusion Principle – combo book 4
3. Spacing Trick – combo book 11
4. Combinations with Repetition – combo book 13, 46

Problems to go with Lecture Topics (from 102 Combinatorics Problems)

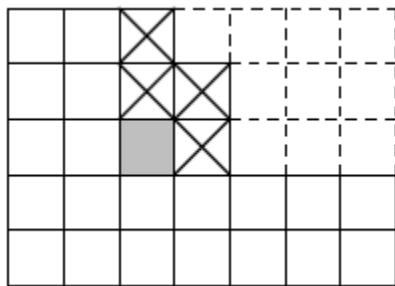
1) Mr. and Mrs. Zeta want to name their baby Zeta so that its monogram (first, middle and last initials) will be in alphabetical order with no letters repeated. How many such monograms are possible?

4) How many positive integers not exceeding 2001 are multiples of 3 or 4 but not 5?

11) Determine the number of ways to choose five numbers from the first eighteen positive integers such that any two chosen numbers differ by at least 2.

13) Find the number of ordered quadruples (x_1, x_2, x_3, x_4) of positive odd integers that satisfy the equation $x_1 + x_2 + x_3 + x_4 = 98$.

46) In a game of *Chomp*, two players alternately take bites from a 5-by-7 grid of unit squares. To take a bite, a player chooses one of the remaining squares, then removes ("eats") all squares in the quadrant defined by the left edge (extended upward) and the lower edge (extended rightward) of the chosen square. For example, the bite determined by the shaded square in the diagram would remove the shaded square and the four squares marked by \times . (The squares with two or more dotted edges have been removed from the original board in previous moves.)



Problems from recent AIMEs

2020 AIME-I 5) Six cards are numbered 1 through 6 are to be lined up in a row. Find the number of arrangements of these six cards where one of the cards can be removed leaving the remaining 5 cards in ascending or descending order.

2020 AIME-I 7) A club consisting of 11 men and 12 women needs to choose a committee from among its members so that the number of women on the committee is one more than the number of men on the committee. The committee could have as few as 1 member or as many as 23 members. Let N be the number of such committees that can be formed. Find the sum of the prime numbers that divide N .

2021 AIME-I 4) Find the number of ways 66 identical coins can be separated into 3 non-empty piles so that there are fewer coins in the first pile than the second pile and fewer coins in the second pile than the third pile.

2021 AIME-II 6) For any finite set S , let $|S|$ denote the number of elements in S . Find the number of ordered pairs (A, B) such that A and B are (not necessarily distinct) subsets of $\{1, 2, 3, 4, 5\}$ that satisfy

$$|A||B| = |A \cap B||A \cup B|$$