

Example Probability Problems Worked Out

Problem (2021 Fall 12A Q 18)

Each of the 20 balls is tossed independently and at random into one of the 5 bins. Let p be the probability that some bin ends up with 3 balls, another with 5 balls, and the other three

with 4 balls each. Let q be the probability that every bin ends up with 4 balls. What is $\frac{p}{q}$?

- (A) 1 (B) 4 (C) 8 (D) 12 (E) 16

Problem 11 (2021 Fall 12B)

Una rolls 6 standard 6-sided dice simultaneously and calculates the product of the 6 numbers obtained. What is the probability that the product is divisible by 4?

- (A) $\frac{3}{4}$ (B) $\frac{57}{64}$ (C) $\frac{59}{64}$ (D) $\frac{187}{192}$ (E) $\frac{63}{64}$

Problem 11 (2020 12A)

A frog sitting at the point $(1, 2)$ begins a sequence of jumps, where each jump is parallel to one of the coordinate axes and has length 1, and the direction of each jump (up, down, right, or left) is chosen independently at random. The sequence ends when the frog reaches a side of the square with vertices $(0, 0)$, $(0, 4)$, $(4, 4)$, and $(4, 0)$. What is the probability that the sequence of jumps ends on a vertical side of the square?

- (A) $\frac{1}{2}$ (B) $\frac{5}{8}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{7}{8}$

Problem 16 (2020 12A)

A point is chosen at random within the square in the coordinate plane whose vertices are $(0, 0)$, $(2020, 0)$, $(2020, 2020)$, and $(0, 2020)$. The probability that the point is within d units of a lattice point is $\frac{1}{2}$. (A point (x, y) is a lattice point if x and y are both integers.) What is d to the nearest tenth?

- (A) 0.3 (B) 0.4 (C) 0.5 (D) 0.6 (E) 0.7

Problem 17 (2021 Fall 12B)

A bug starts at a vertex of a grid made of equilateral triangles of side length 1. At each step the bug moves in one of the 6 possible directions along the grid lines randomly and independently with equal probability. What is the probability that after 5 moves the bug never will have been more than 1 unit away from the starting position?

- (A) $\frac{13}{108}$ (B) $\frac{7}{54}$ (C) $\frac{29}{216}$ (D) $\frac{4}{27}$ (E) $\frac{1}{16}$