OMC Probability Homework Given 10/20/2023

Problem 15 (2010 12 A)

A coin is altered so that the probability that it lands on heads is less than $\frac{1}{2}$ and when the coin is flipped four times, the probability of an equal number of heads and tails is $\frac{1}{6}$. What is the probability that the coin lands on heads?

$\textbf{(A)}\ \frac{\sqrt{15}-3}{6} \qquad \textbf{(B)}\ \frac{6-\sqrt{6\sqrt{6}+2}}{12} \qquad \textbf{(C)}\ \frac{\sqrt{2}-1}{2} \qquad \textbf{(D)}\ \frac{3-\sqrt{3}}{6} \qquad \textbf{(E)}\ \frac{\sqrt{3}-1}{2}$

Problem 16 (2010 12 A)

Bernardo randomly picks 3 distinct numbers from the set $\{1,2,3,...,7,8,9\}$ and arranges them in descending order to form a 3-digit number. Silvia randomly picks 3 distinct numbers from the set $\{1,2,3,...,6,7,8\}$ and also arranges them in descending order to form a 3-digit number. What is the probability that Bernardo's number is larger than Silvia's number?

$\textbf{(A)}\ \frac{47}{72} \qquad \textbf{(B)}\ \frac{37}{56} \qquad \textbf{(C)}\ \frac{2}{3} \qquad \textbf{(D)}\ \frac{49}{72} \qquad \textbf{(E)}\ \frac{39}{56}$

Problem 11 (2010 12 B)

A palindrome between 1000 and $10,000$ is chosen at random. What is the probability that it is divisible by 7?

$\textbf{(A)}\ \frac{1}{10} \qquad \textbf{(B)}\ \frac{1}{9} \qquad \textbf{(C)}\ \frac{1}{7} \qquad \textbf{(D)}\ \frac{1}{6} \qquad \textbf{(E)}\ \frac{1}{5}$

Problem 16 (2010 12B)

Positive integers a, b, and c are randomly and independently selected with replacement from the set $\{1, 2, 3,\dots, 2010\}$. What is the probability that $abc + ab + a$ is divisible by 3?

$\textbf{(A)}\ \frac{1}{3} \qquad \textbf{(B)}\ \frac{29}{81} \qquad \textbf{(C)}\ \frac{31}{81} \qquad \textbf{(D)}\ \frac{11}{27} \qquad \textbf{(E)}\ \frac{13}{27}$

Problem 18 (2010 12B)

A frog makes 3 jumps, each exactly 1 meter long. The directions of the jumps are chosen independently at random. What is the probability that the frog's final position is no more than 1 meter from its starting position?

$\textbf{(A)}\ \frac{1}{6} \qquad \textbf{(B)}\ \frac{1}{5} \qquad \textbf{(C)}\ \frac{1}{4} \qquad \textbf{(D)}\ \frac{1}{3} \qquad \textbf{(E)}\ \frac{1}{2}$

Problem 11 (2012 12A)

Alex, Mel, and Chelsea play a game that has 6 rounds. In each round there is a single winner, and the outcomes of the rounds are independent. For each round the probability that Alex wins is $\frac{1}{2}$, and Mel is twice as likely to win as Chelsea. What is the probability that Alex wins three rounds, Mel wins two rounds, and Chelsea wins one round?

$\textbf{(A)}\ \frac{5}{72} \qquad \textbf{(B)}\ \frac{5}{36} \qquad \textbf{(C)}\ \frac{1}{6} \qquad \textbf{(D)}\ \frac{1}{3} \qquad \textbf{(E)}\ 1$

Problem 15 (2012 12A)

A 3×3 square is partitioned into 9 unit squares. Each unit square is painted either white or black with each color being equally likely, chosen independently and at random. The square is then rotated 90° clockwise about its center, and every white square in a position formerly occupied by a black square is painted black. The colors of all other squares are left unchanged. What is the probability that the grid is now entirely black?

$\textbf{(A)}\ \frac{49}{512} \qquad \textbf{(B)}\ \frac{7}{64} \qquad \textbf{(C)}\ \frac{121}{1024} \qquad \textbf{(D)}\ \frac{81}{512} \qquad \textbf{(E)}\ \frac{9}{32}$

Problem 22 (2013 12A)

A palindrome is a nonnegative integer number that reads the same forwards and backwards when written in base 10 with no leading zeros. A 6-digit palindrome n is chosen uniformly at random. What is the probability that $\frac{n}{11}$ is also a palindrome?

$\textbf{(A)}\ \frac{8}{25} \qquad \textbf{(B)}\ \frac{33}{100} \qquad \textbf{(C)}\ \frac{7}{20} \qquad \textbf{(D)}\ \frac{9}{25} \qquad \textbf{(E)}\ \frac{11}{30}$

Problem 22 (2014 12B)

In a small pond there are eleven lily pads in a row labeled 0 through 10. A frog is sitting on pad 1. When the frog is on pad N, $0 < N < 10$, it will jump to pad $N-1$ with probability $\frac{N}{10}$ and to pad $N+1$ with probability $1-\frac{N}{10}$. Each jump is independent of the previous jumps. If the frog reaches pad 0 it will be eaten by a patiently waiting snake. If the frog reaches pad 10 it will exit the pond, never to return. What is the probability that the frog will escape without being eaten by the snake?

$\textbf{(A) }\frac{32}{79}\qquad \textbf{(B) }\frac{161}{384}\qquad \textbf{(C) }\frac{63}{146}\qquad \textbf{(D) }\frac{7}{16}\qquad \textbf{(E) }\frac{1}{2}$

Problem 17 (2015 12A)

Eight people are sitting around a circular table, each holding a fair coin. All eight people flip their coins and those who flip heads stand while those who flip tails remain seated. What is the probability that no two adjacent people will stand?

$\textbf{(A) }\frac{47}{256}\qquad \textbf{(B) }\frac{3}{16}\qquad \textbf{(C) }\frac{49}{256}\qquad \textbf{(D) }\frac{25}{128}\qquad \textbf{(E) }\frac{51}{256}$

Problem 17 (2015 12B)

An unfair coin lands on heads with a probability of $\frac{1}{4}$. When tossed n times, the probability of exactly two heads is the same as the probability of exactly three heads. What is the value of n ?

$\textbf{(A) }5\qquad \textbf{(B) }8\qquad \textbf{(C) }10\qquad \textbf{(D) }11\qquad \textbf{(E) }13$

Problem 19 (2016 12A)

Jerry starts at 0 on the real number line. He tosses a fair coin 8 times. When he gets heads, he moves 1 unit in the positive direction; when he gets tails, he moves 1 unit in the negative direction. The probability that he reaches 4 at some time during this process is $\frac{a}{b},$ where a and b are relatively prime positive integers. What is $a + b?$ (For example, he succeeds if his sequence of tosses is $HTHHHHHH$.)

$\textbf{(A) }69\qquad \textbf{(B) }151\qquad \textbf{(C) }257\qquad \textbf{(D) }293\qquad \textbf{(E) }313$

Problem 23 (2016 12A)

Three numbers in the interval $[0, 1]$ are chosen independently and at random. What is the probability that the chosen numbers are the side lengths of a triangle with positive area?

$\textbf{(A)}\ \frac{1}{6} \qquad \textbf{(B)}\ \frac{1}{3} \qquad \textbf{(C)}\ \frac{1}{2} \qquad \textbf{(D)}\ \frac{2}{3} \qquad \textbf{(E)}\ \frac{5}{6}$

Problem 19 (2016 12B)

Tom, Dick, and Harry are playing a game. Starting at the same time, each of them flips a fair coin repeatedly until he gets his first head, at which point he stops. What is the probability that all three flip their coins the same number of times?

$\textbf{(A)}\ \frac{1}{8} \qquad \textbf{(B)}\ \frac{1}{7} \qquad \textbf{(C)}\ \frac{1}{6} \qquad \textbf{(D)}\ \frac{1}{4} \qquad \textbf{(E)}\ \frac{1}{3}$