

Orlando Math Circle AMC Prep 9/8/2023

Algebra

- 1) If m and n are different positive integers, and if $\frac{\frac{1}{m} - \frac{1}{n}}{1 - \frac{1}{mn}} = 1$, what is the value of m ?
- 2) If x is a real number for which $\frac{1}{x^3 - 3x^2 + 7x - 5} = \frac{5}{6}$, what is the value of $\frac{1}{x^3 - 3x^2 + 7x - 4}$?
- 3) If $x = t^2 - 4t$ and $y = \frac{1}{t-2}$, what is the value of x when $y = \frac{1}{3\sqrt{2}}$?

Counting

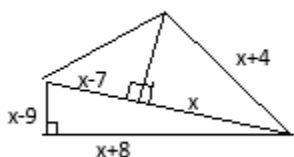
- 1) How many ordered triplets of integers (a, b, c) are there where $0 < a < b < c < 15$?
- 2) How many integers in between 1 and 1000 are divisible by 2, 3 or 5?
- 3) An animal shelter has two cats: Felix and Flopsy and three dogs: Roger, Amelia and Coco. All of these animals are to be adopted by a potential four owners: Monty, Nellie, Ophelia and Paul. Assuming that no owner receives both a cat and a dog and that it's not necessary for each potential owner to receive a pet, in how many ways can the five pets be distributed amongst the owners? (Two arrangements are different if any owner-pet combo is different between the two arrangements. For example, if Monty owns Felix in one arrangement but Nellie owns her in another, those two are different arrangements.)

Number Theory

- 1) p and q are distinct primes and $50!$ is divisible by both p and q . What is the maximum possible value of pq ?
- 2) n is a positive integer with exactly 15 divisors. What is the smallest possible value of n ?
- 3) Define $f(n)$ to be the least common multiple of the positive integers 1 through n . For example $f(4) = 12$ and $f(5) = 60$. Under what circumstances does $f(n) = f(n-1)$?

Geometry

- 1) ABCDEF is a regular hexagon. What is the ratio of the area of BDF (an equilateral triangle formed with alternating vertices of the hexagon) to the area of ABCDEF?
- 2) In the quadrilateral shown, one of the diagonals is drawn, some of the line segments are marked with their lengths, and some line segments are marked as being perpendicular. (There are three right angled triangles in the picture.) What is the perimeter of the quadrilateral?



- 3) Equilateral Gothic arch ABC is made by drawing line segment AC, circular arc AB with center C, and circular arc BC with center A. A circle inscribed in this Gothic arch is tangent to arc AB, arc BC and line segment AC. If $AC = 40$, what is the area of the circle?

Logs

- 1) Determine the value of $x > 1$ for which $\log_2(2 + x) = \log_x 2 + \log_x x$.
- 2) Find both values of x which satisfy the following equation:

$$\frac{\log_2 x}{\log_4 2x} = \frac{\log_8 4x}{\log_{16} 8x}$$

- 3) Define a function f as follows: $f(1) = 2$. For all integers $n > 1$, $f(n) = (f(n - 1))^{2^n}$.

What is the value of $\log_{65536}(\log_{65536}(f(4)))$?

Express your answer in the form $2^a + 2^b$, where both a and b are integers. Note that $65536 = 2^{16}$.