OMC Practice Questions for AMC Prep

Set #1 D = RT
1) Jeslyn runs a five mile race. She completes the first two miles running at an average rate of 4 miles per hour. It takes her 48 minutes to complete the last three miles of the race. What was her average speed for the whole five mile race, in miles per hour? Please leave your answer as a reduced fraction (in the form p/q where both p and q are positive integers that don't share a common factor.)

2) An shuttle bus is making a 200 mile trip. For the first portion of the trip, the bus averages 60 miles an hour. Unfortunately, a rain storm hits and for the rest of the trip (second portion), the bus averages 48 miles per hour. The average speed of the whole 200 mile trip was 50 miles an hour. How long, **in miles**, was the first portion of the trip?

3) The current in a river is flowing steadily at 3 miles per hour. A motor boat which travels at a constant rate in still water goes downstream 4 miles and then returns to its starting point. The trip takes one hour, excluding the time spent in turning the board around. What is the ratio of the downstream to the upstream rate?

Set #2 Probability
1) There are 20 marbles in a bag. 8 of the marbles are blue. Megan randomly selects 2 of the marbles at one time. What is the probability both are blue?

2) Alex, Mel, and Chelsea play a game that has 6 rounds. In each round there is a single winner, and the outcomes of the rounds are independent. For each round the probability that Alex wins is $\frac{1}{2}$, and Mel is twice as likely to win as Chelsea. What is the probability that Alex wins three rounds, Mel wins two rounds, and Chelsea wins one round?

3) Eight people are sitting around a circular table, each holding a fair coin. All eight people flip their coins and those who flip heads stand while those who flip tails remain seated. What is the probability that no two adjacent people will stand?

Set #3 Logs
1) For how many integral values of x can a triangle of positive area be formed having side lengths of $\log_2 x$, $\log_4 x$, and 3?

2) What is the value of the following summation: $\log_{n!} 1 + \log_{n!} 2 + \log_{n!} 3 + \cdots \log_{n!} n$?

3) Positive real numbers x ≠ 1, y ≠ 1, satisfy $\log_2 x = \log_y 16$ and $xy = 64$. What is $(\log_2 \frac{x}{y})^2$?

Set #4 Counting

1) How many permutations are there of the letters in the word CATTRAP? Simplify your answer to a single integer.

2) At a twins and triplets convention, there were 9 sets of twins and 6 sets of triplets, all from different families. Each twin shook hands with all the twins except his/her sibling and with half the triplets. Each triplet shook hands with all the triplets except his/her siblings and with half the twins. How many handshakes took place?

3) How many ordered quadruplets (a_1, a_2, a_3, a_4) of non-negative integers, where at least one of the integers is even, satisfy the equation $a_1 + a_2 + a_3 + a_4 = 100$? Please express your answer in the form $\binom{w}{x} - \binom{y}{z}$. (Note that the values of w, x, y and z will be integers, but not necessarily all distinct.)

Set #5 Divisors, GCD, LCM

1) How many positive integers less than 500 have an odd number of divisors?

2) Let M be the least common multiple of all the integers 10 through 30, inclusive. Let N be the least common multiple of M, 32, 33, 34, 35, 36, 37, 38, 39 and 40. What is the value of $\frac{N}{M}$?

3) A positive integer n has 60 divisors and 7n has 80 divisors. What is the greatest integer k such that 7^k divides n?