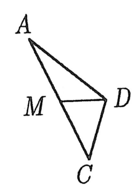
Thus, $EF/FC = (EC/3)/(2EC/3) = 1/2$. Putting this together with AF/FD, we find $EF/FC + AF/FD = \mathbf{3/2}$.

This is a very difficult problem, but it shows the amazing amount of information that can be found from cleverly adding a single segment to a diagram.

Problems to Solve for Chapter 11

167. In $\triangle ADC$, segment DM is drawn such that $\angle ADM = \angle ACD$. Prove that $AD^2 = (AM)(AC)$.



168. How many scalene triangles have all sides of integral lengths and perimeter less than 13? (AHSME 1956)

169. The sides of $\triangle BAC$ are in the ratio $2:3:4$. BD is the angle bisector drawn to the shortest side AC, dividing it into segments AD and CD. If the length of AC is 10, then find the length of the longer segment of AC. (AHSME 1966)

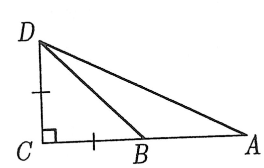
170. What is the number of distinct lines representing the altitudes, medians, and interior angle bisectors of a triangle that is isosceles, but not equilateral? (AHSME 1957)

171. Triangle ABD is right-angled at B. On AD there is a point C for which $AC = CD$ and $AB = BC$. Find $\angle DAB$. (AHSME 1963)

172. Triangle PYT is a right triangle in which $PY = 66$ and $YT = 77$. If PT is more than 50 and is expressed in the simplified form $x\sqrt{y}$, then find $x + y$. (MAΘ 1990)

173. If triangle PQR has sides 40, 60, and 80, then the shortest altitude is K times the longest altitude. Find the value of K. (MATHCOUNTS 1990)

174. In this figure, $\angle ACD$ is a right angle, A, B, and C are collinear, $\angle A = 30°$, and $\angle DBC = 45°$. If $AB = 3 - \sqrt{3}$, find the area of $\triangle BCD$. (MAΘ 1992)



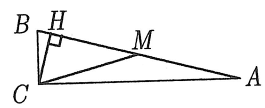
175. The perpendicular bisectors of two of the sides of triangle ABC intersect the third side at the same point. Prove that the triangle is right-angled. (M&IQ 1992)

176. Show that if h_a, h_b, and h_c are the altitudes of a triangle, then

$$\frac{1}{h_a} < \frac{1}{h_b} + \frac{1}{h_c}.$$

177. In right triangle ABC, $\angle C = 90°$ and $\sin A = 7/25$. Find $\sin B$, $\cos A$, $\cot A$, and $\csc B$.

178. The angle between the median CM and the hypotenuse AB of right triangle ABC is equal to $30°$. Find the area of ABC if the altitude CH is equal to 4. (M&IQ 1992)



179. The base of a triangle is 15 inches. Two lines are drawn parallel to the base, terminating in the other two sides and dividing the triangle into three equal areas. What is the length of the parallel closer to the base? (AHSME 1953)

180. The straight line AB is divided at C so that $AC = 3CB$. Circles are drawn with AC and CB as diameters and a common tangent to these meets AB extended at D. Show that BD equals the radius of the smaller circle. (AHSME 1954)

181. Segments AD and BE are medians of right triangle ABC, and AB is its hypotenuse. If a right triangle is constructed with legs AD and BE, what will be the length of its hypotenuse in terms of AB? (Mandelbrot #2)

182. Let CM be the median in equilateral triangle ABC. Point N is on BC such that $MN \perp BC$. Prove that $4BN = BC$. (M&IQ 1992)

183. In right triangle ACD with right angle at D, B is a point on side AD between A and D. The length of segment AB is 1. If $\angle DAC = \alpha$ and $\angle DBC = \beta$, then find the length of side DC in terms of α and β. (MAΘ 1991)

184. Angle B of $\triangle ABC$ is trisected by BD and BE which meet AC at D and E respectively. Prove that

$$\frac{AD}{EC} = \frac{(AB)(BD)}{(BE)(BC)}.$$

(AHSME 1952)

185. Given that I is the incenter of $\triangle ABC$, $AB = AC = 5$, and $BC = 8$, find the distance AI. (Mandelbrot #3)