Engineering Analysis ENG 3420 Fall 2009

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Lecture 17

- Reading assignment Chapters 10 and 11, Linear Algebra ClassNotes
- Last time:
  - Symmetric matrices; Hermitian matrices.
  - Matrix multiplication
- Today:
  - Linear algebra functions in Matlab
  - The inverse of a matrix
  - Vector products
  - Tensor algebra
  - Characteristic equation, eigenvectors, eigenvalues
  - Norm
  - Matrix condition number
- Next Time
  - More on
    - LU Factorization
    - Cholesky decomposition

Matrix analysis in MATLAB

- Norm → Matrix or vector norm
- normest → Estimate the matrix 2-norm
- rank → Matrix rank
- det → Determinant
- trace → Sum of diagonal elements
- null → Null space
- orth → Orthogonalization
- rref → Reduced row echelon form
- subspace → Angle between two subspaces

Eigenvalues and singular values

- eig → Eigenvalues and eigenvectors
- svd → Singular value decomposition
- eigs → A few eigenvalues
- svds → A few singular values
- poly → Characteristic polynomial
- polyeig → Polynomial eigenvalue problem
- condeig → Condition number for eigenvalues
- hess → Hessenberg form
- qz → QZ factorization
- schur → Schur decomposition

Matrix functions

Expm → Matrix exponential

Logm → Matrix logarithm

Sqrtm → Matrix square root

Funm → Evaluate general matrix function

Linear systems of equations

\ and / → Linear equation solution
inv → Matrix inverse
cond → Condition number for inversion
condest → 1-norm condition number estimate
chol → Cholesky factorization
cholinc → Incomplete Cholesky factorization
linsolve → Solve a system of linear equations
lu → LU factorization
ilu → Incomplete LU factorization
luinc → Incomplete LU factorization
qr → Orthogonal-triangular decomposition
lsqnonneg → Nonnegative least-squares
pinv → Pseudoinverse
lscov → Least squares with known covariance

Distance and norms

- Metric space → a set where the ”distance” between elements of the set is defined, e.g., the 3-dimensional Euclidean space. The Euclidean metric defines the distance between two points as the length of the straight line connecting them.

- A *norm* → real-valued function that provides a measure of the size or “length” of an element of a vector space.

Vector Norms

- The *p*-norm of a vector X is: $\|X\|_p = \left(\sum_{i=1}^{n} |x_i|^p \right)^{1/p}$

- Important examples of vector *p*-norms include:

$$p = 1 : \text{sum of the absolute values} \quad \|X\|_1 = \sum_{i=1}^{n} |x_i|$$

$$p = 2 : \text{Euclidian norm (length)} \quad \|X\|_2 = \|X\|_e = \sqrt{\sum_{i=1}^{n} x_i^2}$$

$$p = \infty : \text{maximum} - \text{magnitude} \quad \|X\|_\infty = \max_{1 \le i \le n} |x_i|$$

Matrix Norms

- Common matrix norms for a matrix [*A*] include:

column - sum norm $\quad \|A\|_1 = \max_{1 \le j \le n} \sum_{i=1}^{n} |a_{ij}|$

Frobenius norm $\quad \|A\|_f = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^2}$

row - sum norm $\quad \|A\|_\infty = \max_{1 \le i \le n} \sum_{j=1}^{n} |a_{ij}|$

spectral norm (2 norm) $\quad \|A\|_2 = (\mu_{\max})^{1/2}$

- Note - μ_{max} is the largest eigenvalue of $[A]^T[A]$.

Matrix Condition Number

- The *matrix condition number* Cond[A] is obtained by calculating Cond[A]=$||A||\cdot||A^{-1}||$
- In can be shown that:

$$\frac{\|\Delta X\|}{\|X\|} \leq \mathrm{Cond}[A]\frac{\|\Delta A\|}{\|A\|}$$

- The relative error of the norm of the computed solution can be as large as the relative error of the norm of the coefficients of [A] multiplied by the condition number.
- If the coefficients of [A] are known to t digit precision, the solution [X] may be valid to only t-$\log_{10}$(Cond[A]) digits.

Built-in functions to compute norms and condition numbers

- norm(*X*,*p*) → Compute the *p* norm of vector *X*, where *p* can be any number, *inf*, or *'fro'* (for the Euclidean norm)
- norm(*A*,*p*) → Compute a norm of matrix *A*, where *p* can be 1, 2, *inf,* or *'fro'* (for the Frobenius norm)
- cond(*X*,*p*) or cond(*A*,*p*) → Calculate the condition number of vector *X* or matrix *A* using the norm specified by *p*.