Engineering Analysis ENG 3420 Fall 2009

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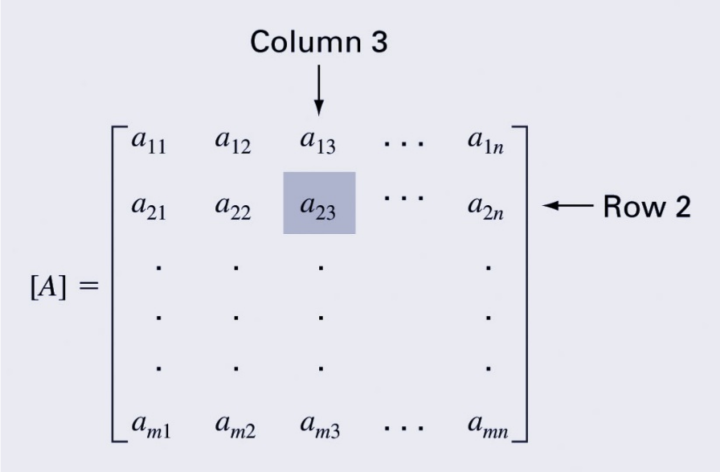
Lecture 12

- Last time:
  - Optimization
    - Golden ratio → makes one-dimensional optimization efficient.
    - Parabolic interpolation →locate the optimum of a single-variable function.
    - fminbnd function → determine the minimum of a one-dimensional function.
    - fminsearch function → determine the minimum of a multidimensional function.
  - Heuristics for solving search/optimization problems
- Today:
  - Problem solving in preparation for the quiz
  - Linear Algebra Concepts
    - Vector Spaces, Linear Independence
    - Orthogonal Vectors, Bases
    - Matrices
- Next Time
  - Gauss elimination

Matrices

- M*atrix* → a rectangular array of elements, e.g., [A]. An *element* of the matrix e.g., $a_{23.}$
- The size of a matrix is given as
  m rows by *n* columns, or simply
  m by *n* (or *m* x *n*).
- 1 x *n* matrices are *row vectors*.
- *m* x 1 matrices are *column vectors.*



$$[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

Special Matrices

- Matrices where *m=n* are called *square matrices*.
- There are a number of special forms of square matrices:

Symmetric

$$[A]=\begin{bmatrix} 5 & 1 & 2 \\ 1 & 3 & 7 \\ 2 & 7 & 8 \end{bmatrix}$$

Diagonal

$$[A]=\begin{bmatrix} a_{11} & & \\ & a_{22} & \\ & & a_{33} \end{bmatrix}$$

Identity

$$[A]=\begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$

Upper Triangular

$$[A]=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ & a_{22} & a_{23} \\ & & a_{33} \end{bmatrix}$$

Lower Triangular

$$[A]=\begin{bmatrix} a_{11} & & \\ a_{21} & a_{22} & \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Banded

$$[A]=\begin{bmatrix} a_{11} & a_{12} & & \\ a_{21} & a_{22} & a_{23} & \\ & a_{32} & a_{33} & a_{34} \\ & & a_{43} & a_{44} \end{bmatrix}$$

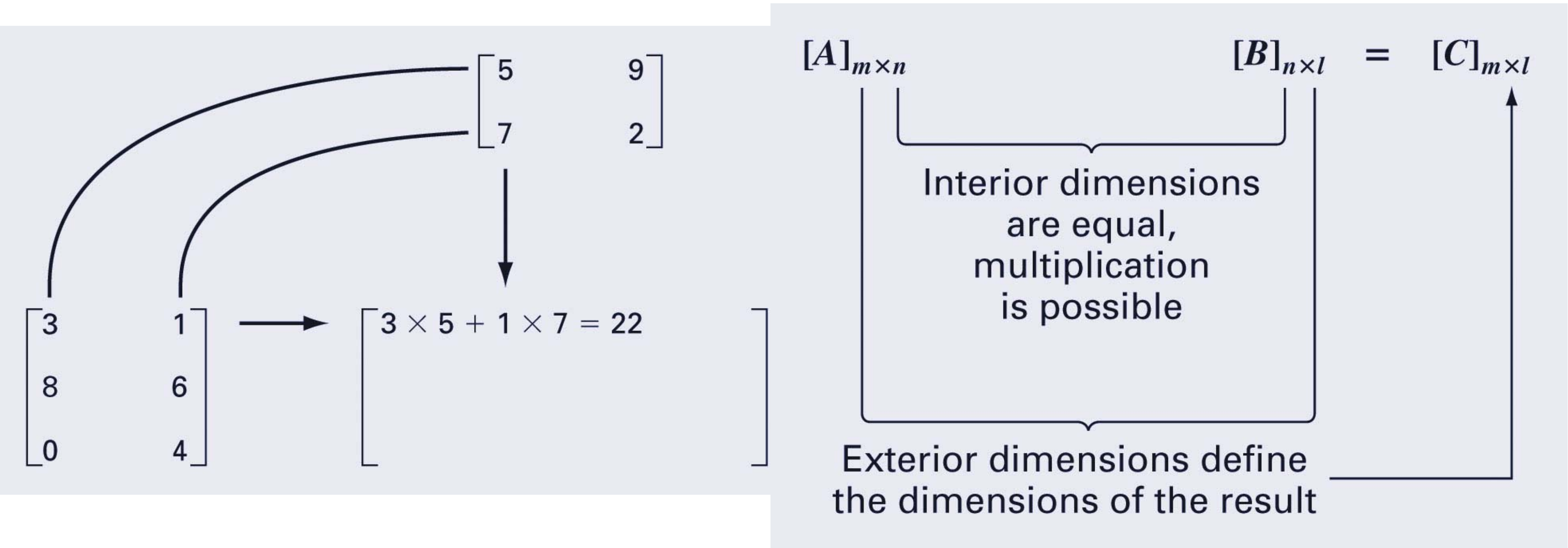
Matrix Operations

- Two matrices are considered equal if and only if every element in the first matrix is equal to every corresponding element in the second. This means the two matrices must be the same size.
- Matrix addition and subtraction are performed by adding or subtracting the corresponding elements. This requires that the two matrices be the same size.
- Scalar matrix multiplication is performed by multiplying each element by the same scalar.

Matrix Multiplication

- The elements in the matrix [C] that results from multiplying matrices [A] and [B] are calculated using:

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$



Inverse and transpose of a matrix

- The *inverse* of a square, nonsingular matrix $[A]$ is that matrix which, when multiplied by $[A]$, yields the identity matrix.
  - $[A][A]^{-1}=[A]^{-1}[A]=[I]$
- The *transpose* of a matrix involves transforming its rows into columns and its columns into rows.
  - $(a_{ij})^T=a_{ji}$

Solving systems of linear equations

- Matrices provide a concise notation for representing and solving simultaneous linear equations:

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3
\end{aligned}
$$

$$
\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}
\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}
=
\begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix}
$$

$$
[A]\{x\} = \{b\}
$$

Solving systems of linear equations in Matlab

- Two ways to solve systems of linear algebraic equations [A]{x}={b}:
  - Left-division

    ```
    x = A\b
    ```
  - Matrix inversion

    ```
    x = inv(A)*b
    ```
- Matrix inversion only works for square, non-singular systems; it is less efficient than left-division.