

University of Central Florida
 School of Electrical Engineering and Computer Science
 EGN-3420 - Engineering Analysis.
 Fall 2009 - dcm

Project 2 due Thursday week 9 (100 points)

In this project you are asked to write a library for tensor operations. The tensor product of an $m \times n$ matrix and a $p \times q$ matrix is an $mp \times nq$ matrix. For example, if A is an $m \times n$ matrix and B is a $p \times q$ matrix then, using the Kronecker product representation for the tensor product, we have

$$A \otimes B = \begin{pmatrix} a_{11}B & a_{12}B & \dots & a_{1n}B \\ a_{21}B & a_{22}B & \dots & a_{2n}B \\ a_{31}B & a_{32}B & \dots & a_{3n}B \\ \vdots & \vdots & & \vdots \\ a_{m1}B & a_{m2}B & \dots & a_{mn}B \end{pmatrix}$$

with

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ a_{31} & a_{32} & \dots & a_{3n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \quad B = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1q} \\ b_{21} & b_{22} & \dots & b_{2q} \\ b_{31} & b_{32} & \dots & b_{3q} \\ \vdots & \vdots & & \vdots \\ b_{p1} & b_{p2} & \dots & b_{pq} \end{pmatrix}.$$

Here, $a_{ij}B$, $1 \leq i \leq m$, $1 \leq j \leq n$ is a sub-matrix whose entries are the products of a_{ij} and all the elements of matrix B .

The tensor product of two vectors, one of dimension p , and the other of dimension q is a vector of dimension pq . For example, the tensor product of two-dimensional vectors $(a, b)^t$ and $(c, d)^t$ is the four-dimensional vector:

$$\begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix}.$$

The tensor product of two vectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is:

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}.$$