University of Central Florida
School of Electrical Engineering and Computer Science
EGN-3420 - Engineering Analysis.
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**Project 1 due Thursday week 6 (100 points)**

Optimization methods Optimization problems require the determination of the extremes (maxima or minima) of continuous functions of one or more variables. One of the methods to locate the value of the argument $x$ of a function $f(x)$ which maximizes or minimizes the value of the function is the *golden section*.

If $x_m$ is an extreme of the function $f(x)$ with a derivative $f'(x)$ then $f'(x = x_m) = 0$. Therefore, there should be no surprise that finding the extreme of a function $f(x)$ is related to the problem of finding the roots of the equation $f'(x) = 0$ for $x \in (x_{L_0}, s_{H_0})$. The so-called *bisection method* for finding the roots divides the interval $I_0 = (x_{L_0}, x_{H_0})$ in half, recursively; if in this process the function $g(x)$ changes signs in the interval $I_k = (x_{L_k}, x_{H_k})$ then we evaluate $g(x = x_k)$ with $x_k = (x_{H_k} - x_{L_k})/2$. The new search interval becomes the subinterval where $g(x)$ changes sign, see Figure 1 (a).

Now we discuss discuss Euclid's condition used to define the golden number $\varphi$. Given a segment of length $y_1 + y_2$ Figure 1 (b) the condition is:

$$\left(\frac{y_1}{y_2}\right)^2 = \frac{y_1 + y_2}{y_2}$$

If we use the notation $\varphi = y_1/y_2$ this condition leads to the equation $\varphi^2 - \varphi - 1 = 0$ with a positive root $\varphi = (1 + \sqrt{5})/2 = 1.680133$.

The idea of the golden section is similar with the bisection yet, instead of using a single intermediate value (the mid point of the interval) we use two intermediate points $x_1$ and $x_2$ determined using . If the original interval is $I_0 = (x_{L_0}, x_{H_0})$ then we compute:

$$x_1 = x_{L_0} + d \qquad \text{and} \qquad x_2 = x_{H_0} - d \qquad \text{with} \qquad d = (\varphi - 1)(x_{H_0} - x_{L_0}).$$

For the case in Figure 1 (c) we see that $f(x_1) > f(x_2)$; this means that the minimum is to the right of $f(x_2)$, thus the new search interval is $x_{L_1} = x_2$ and $x_{H_1} = x_{H_0}$. The process continues recursively.

Task 1 (20 points) Construct a Matlab function which determines the maximum of a function $f(x)$ in the interval $I_0 = (x_{L_0}, x_{H_0})$ using the golden-section method. Use as a model the algorithm to determine the minimum of a function in Figure 7.7 in the textbook; do not simply apply this algorithm to find the minimum of the function $-f(x)$. The algorithm should have as input:

- the name of a function;
- the lower and the upper limits of the search interval;
- the maximum allowable number of iterations; and
- the desired relative error.

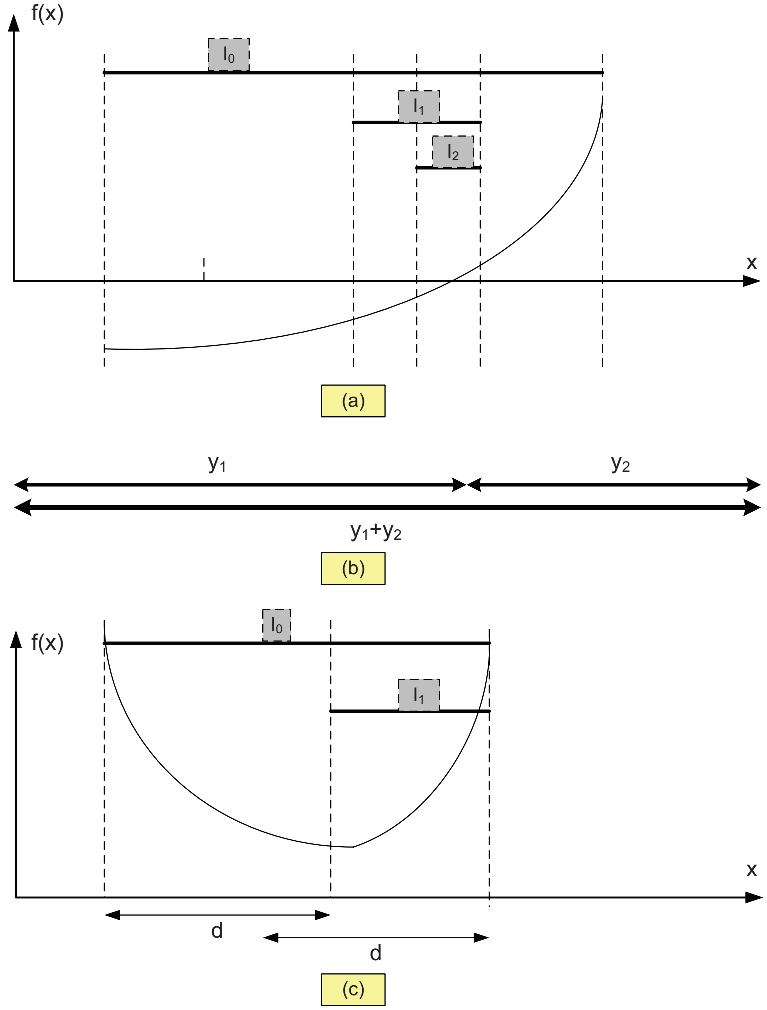


Figure 1: (a) The bisection method. (b) Euclid's condition $\frac{y_1}{y_2} = \frac{y_1+y_2}{y_2}$. (c) The golden rule. If the original interval is $I_0 = (x_{L_0}, x_{H_0})$ then we compute: $d = (\varphi - 1)(x_{H_0} - x_{L_0})$, $x_1 = x_{L_0} + d$, and $x_2 = x_{H_0} - d$. We notice that $f(x_1) > f(x_2)$; this means that the minimum is to the right of $f(x_2)$, thus the new search interval is $I_1 = (x_{L_1}, x_{H_1})$ with $x_{L_1} = x_2$ and $x_{H_1} = x_{H_0}$ .

The output of the function should be:

- the maximum of the function
- the argument for which the function reaches the maximum;
- the actual number of iterations; and
- the actual relative error.

Test your function with: (i) the function $f(x) = 2\sin x - x^2/10$ with initial guesses $x_1 = 0$ and $x_2 = 4$; (ii) the function used in Example 7.1 on page 168 of the textbook.

Task 2 (20 points) Plot the following function: $f(x,y) = x - y + (x-y)^2 + (x+y)^2 + 4(1+xy)$ using contour and mesh plots for $x, y \in (-10, +10)$. Attempt to locate on the graphs the extreme points.

Task 3 (20 points) Study and report the algorithm based upon the Nedler-Mead method used by the Matlab function *fminsearch* to determine the minimum of a function. The Nelder-Mead method or downhill simplex method or amoeba method is a commonly used nonlinear optimization technique.

Task 4 (20 points) Plot the function $f(x, y) = 5x^2 - 5xy + 2.5y^2 - x - 1.5y$ and use the Matlab function *fminsearch* to locate the values of $x$ and $y$ that minimize the function. $f(x, y)$ in this problem is a finite-element model of a cantilever. A cantilever is a beam supported on only one end; the beam carries the load to the support where it is resisted by moment and shear stress.

Task 5 (20 points) Construct a Matlab function which determines the minimum of a function $f(x)$ in the interval $I_0 = (x_{L_0}, x_{H_0})$ using the parabolic method. Test your program using the problem in Example 7.3 on page 177 of the textbook.