Worm Propagation Modeling and Analysis under Dynamic Quarantine Defense

Cliff Changchun Zou
Dept. Electrical & Computer Engineering
Univ. Massachusetts
Amherst, MA
czou@ecs.umass.edu

Weibo Gong
Dept. Electrical & Computer Engineering
Univ. Massachusetts
Amherst, MA
gong@ecs.umass.edu

Don Towsley
Dept. Computer Science
Univ. Massachusetts
Amherst, MA
towsley@cs.umass.edu

ABSTRACT
Due to the fast spreading nature and great damage of Internet worms, it is necessary to implement automatic mitigation, such as dynamic quarantine, on computer networks. Enlightened by the methods used in epidemic disease control in the real world, we present a dynamic quarantine method based on the principle “assume guilty before proven innocent” — we quarantine a host whenever its behavior looks suspicious by blocking traffic on its anomaly port. Then we will release the quarantine after a short time, even if the host has not been inspected by security staffs yet. We present mathematical analysis of three worm propagation models under this dynamic quarantine method. The analysis shows that the dynamic quarantine can reduce a worm’s propagation speed, which can give us precious time to fight against a worm before it is too late. Furthermore, the dynamic quarantine will raise a worm’s epidemic threshold, thus it will reduce the chance for a worm to spread out. The simulation results verify our analysis and demonstrate the effectiveness of the dynamic quarantine defense.

Categories and Subject Descriptors
K.6.5 [Management of computing and information systems]: Security and Protection—Invasive software

General Terms
Security

Keywords
dynamic quarantine, worm propagation, epidemic model

1. INTRODUCTION
Since the Morris worm in 1988 [9], the security threat posed by worms has steadily increased, especially in the last several years. In 2001, the Code Red and Nimda infected hundreds of thousands computers [10] [13], cost millions of dollars loss to our society [16]. In January 2003, the SQL Slammer worm spread out and infected more than 90% of vulnerable computers within 10 minutes [12]. Fortunately, none of them destroyed information on compromised hosts. However, we cannot depend on the kindness of hackers in the future. These worms have demonstrated how dangerous and how fast a worm can spread to infect almost all vulnerable computers on the Internet before human can take effective counteractions. As the bandwidth of Internet connections keeps increasing, future worms will require even less time to finish the infection task.

For those fast spreading worms, human’s manual counteractions cannot catch up with the worms’ propagation speed. Automatic mitigation is necessary for defending against fast spreading worms in the future. Currently, the popular Intrusion Prevention System (IPS) [8] on the security market can be thought as a product combining intrusion detection with primary automatic mitigation techniques.

Automatic mitigation is not very difficult for known worms. Firewalls or routers can inspect packet contents according to the signatures of known worms. A worm’s packets can be dropped automatically when firewalls or routers find out the signature of the worm. However, no signature is available for an unknown worm — we have to rely on behavior-based anomaly detection methods to detect an unknown worm. The great challenge for automatic mitigation now is that the current behavior-based anomaly detection methods have the common problem of having high false alarm rate. If we rely on automatic mitigation to block an unknown worm, it will also block many legitimated connections or healthy computers. If we release the block on an alarmed host only after security staffs check and find out that the host is healthy, then many innocent healthy hosts will be blocked too long due to human’s slow manual inspection.

Then how can we use current imperfect anomaly detection systems to build an automatic mitigation defense against fast spreading worms? Enlightened by the methods used in epidemic disease control in the real world, we present a dynamic quarantine method based on the principle “assume guilty before proven innocent”. This dynamic quarantine method can alleviate the negative impact of false alarms generated by worm anomaly detection systems.

We quarantine a host whenever its behavior looks suspicious, and release the quarantine automatically after a short
time. If the worm anomaly detection program we use in the system can determine which service port has suspicious activities, then the quarantine means we only block traffic on the suspicious port without interfering normal connections on other ports. Once some hosts give alarms and are quarantined, security staffs should inspect these quarantined hosts as quickly as possible. However, in order not to severely interfere normal activities, the quarantine on a host will be released automatically after a short time, even if the host has not been inspected by security staffs yet. In this way, a falsely quarantined healthy host will not be blocked too long.

We emphasize that this paper is not about how to improve anomaly detection systems. The dynamic quarantine method we present here can be built on any worm anomaly detection systems, where the detection systems are assumed to have certain false positive and false negative.

As a first step in this direction, in this paper we study the case where the quarantine time and the threshold of the worm anomaly detection are constants. We mathematically analyze a worm’s propagation under such dynamic quarantine and present worm models extended from two traditional epidemic models.

1.1 Related Work

In the area of virus and worm modeling, Kephart, White, and Chess of IBM have performed a series of studies on viral infection based on epidemiology models [3, 4, 5]. Staniford et al. use the classical simple epidemic model to model the spread of Code Red [14]. The epidemic model matches well the increasing part of observed Code Red data. Zou et al. present a “two-factor” worm model that considers the effect of human countermeasures and the congestions caused by worm scan traffic [18]. Chen et al. present a discrete-time worm model that considers the patching and cleaning effect during worm propagation [1].

People have studied how to defend against worm propagation, especially after the Code Red incident in 2001. La Brea project attempts to slow down the growth of TCP-based worms by intercepting worm probes to unused IP addresses and putting those connections in a persistent state [7]. However, it can easily be circumvented by a future worm by asynchronously operating the TCP connections. Williamson presents a soft blocking method to restrict the high speed probing rate of infected hosts [17]. This soft blocking method exploits the behavior differences between a normal host and an infectious host: an infectious host will try to connect to many “new” hosts as fast as possible. By restraining the connection rate to new hosts, Williamson’s method can constrain the probing rate of infected hosts and at the same time does not affect much of the normal connections of healthy hosts. Kreidl et al. present a feedback control host-based autonomic defense system to protect the information and functionality of a server [6]. However, their system is mainly about how to detect a worm’s process that is already running on a computer and then recover the computer from the worm. It cannot protect a computer from being infected at the first place. Zou et al. present a non-threshold based worm early detection system by using the idea “detecting the trend, not the rate” of monitored scan traffic [19]. They do not discuss, however, how to deal with false alarms and how to incorporate their system with automatic mitigation.

Moore et al. study the effect of quarantine on the Internet level to constrain worm propagation [11]. They show that an infectious host has many paths to a target due to the high connectivity of the Internet — it will be very challenging to build a quarantine system that can prevent the widespread of a worm on the Internet level. Because an enterprise has the need to protect its own network from worms, and also because security staffs have control over an enterprise network, Silicon Defense company has focused on automatic mitigation on an enterprise-level network. Its “CounterMalice” devices can divide a large enterprise network into many separated subnetworks and automatically block a worm’s traffic when the “CounterMalice” devices detect the worm [15]. In this way, the quarantine of a subnetwork will stop an infectious host in this subnetwork from infecting hosts in other subnetworks of this enterprise network.

The rest of the paper is organized as follows. Section 2 gives brief introduction of two traditional worm propagation models. In Section 3, we present our dynamic quarantine method and mathematically analyze its behavior. In Section 4, we present three worm propagation models for the dynamic quarantine system based on traditional models introduced in Section 2. Then in Section 5, we use simulation to study the performance of the dynamic quarantine system and to verify our analysis. Finally, Section 6 concludes the paper.

2. TRADITIONAL WORM PROPAGATION MODEL

Computer viruses and worms are similar to biological viruses in their self-replicating and propagation behaviors. Thus the mathematical techniques developed for the study of biological infectious diseases can be adapted to the study of computer viruses and worms propagation.

In the epidemiology area, both stochastic models and deterministic models exist for modeling the spread of infectious diseases [2]. Stochastic models are suitable for small-scale systems with simple virus dynamics; deterministic models are suitable for large-scale systems under the assumption of mass action, relying on the law of large number [2]. When we model an Internet worm’s propagation, we consider a large-scale network with thousands of computers. Thus we will only consider deterministic models in this paper. In this section, we first introduce two classical deterministic epidemic models, which have been widely used by many researchers to study Internet worm propagation [5, 11, 14, 18, 19].

In epidemiology modeling, hosts that are vulnerable to a disease are called susceptible hosts; hosts that have been infected and can infect others are called infectious hosts; hosts that are immune or dead such that they can’t be infected by the disease are called removed hosts. In this paper, we will use the same terminology for computer worm modeling.

In this paper, the system under consideration only consists of hosts that are either vulnerable or infected at the beginning of a worm’s propagation. In other words, we ignore all other hosts that have no relationship with the worm under consideration (they do not affect the worm’s spreading). For example, for Code Red worm on July 19th, 2001, the system that we consider consists of all those on-line Windows machines that had the IIS vulnerability right before the worm spread out.
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### 2.1 Simple Epidemic Model

The simple epidemic model assumes that each host stays in one of two states: **susceptible** or **infectious**. The model further assumes that once a host is infected by a virus, it will stay in the infectious state forever. Thus a host can only have one possible state transition: “susceptible → infectious” [2]. Denote $I(t)$ the number of infectious hosts at time $t$; $N$ the number of hosts in the system; $S(t) = N - I(t)$ the number of susceptible hosts at time $t$.

The model assumes that the system is homogeneous — each host has the equal probability to contact any other hosts. Thus the number of contacts between infectious hosts and susceptible hosts is proportional to $S(t)I(t)$. Based on this phenomenon, the classical simple epidemic model for a finite population is

$$
\frac{dI(t)}{dt} = \beta I(t)S(t) = \beta I(t)[N - I(t)],
$$

where $\beta$ is called the **pairwise rate of infection** [2]. At $t = 0$, $I(0)$ hosts are infectious and the other $S(0) = N - I(0)$ hosts are all susceptible.

We define

$$
\alpha = \beta N
$$

as a **worm’s infection rate**. It is the average number of probes an infectious host can send out to the population $N$ during a unit time (the number of probes sent out by an infectious host to the whole Internet can be much larger).

### 2.2 General Epidemic Model: Kermack-McKendrick Epidemic Model

Kermack-McKendrick model considers the removal process of infectious hosts [2]. It assumes that during an epidemic of a contagious disease, some infectious hosts either recover or die, and thus they are immune to the disease forever — the hosts are in **removed** state. Therefore, in this model each host stays in one of three states at any time: **susceptible**, **infectious**, or **removed**. A host either makes the state transition “susceptible → infectious → removed” or remains in “susceptible” state forever.

Denote $U(t)$ the number of removed hosts from previously infected ones at time $t$. Based on the simple epidemic model (1), the Kermack-McKendrick model is

$$
\begin{align*}
\frac{dI(t)}{dt} &= \beta I(t)S(t) - \gamma I(t) \\
\frac{dU(t)}{dt} &= \gamma I(t) \\
N &= I(t) + U(t) + S(t)
\end{align*}
$$

where $\gamma$ is the **removal rate** of infectious hosts.

Define

$$
\rho \equiv \gamma / \beta.
$$

An important result from the Kermack-McKendrick model is the **epidemic threshold theorem**: a major outbreak occurs if and only if the initial number of susceptible hosts $S(0) > \rho$. For this reason, We call $\rho$ as **epidemic threshold** in this paper. The theorem is not hard to understand: from (3), we can derive $dI(t)/dt < 0, \forall t > 0$ if and only if $S(0) < \rho$.

### 3. DYNAMIC QUARANTINE AND ITS ANALYSIS

In automatic mitigation of an Internet worm, when a host is found to be infected, it can immediately be isolated (quarantined) by the worm detection program within seconds or milliseconds. In this way, the defense actions can catch up with a worm’s fast infection speed and constrain the worm’s propagation. For an unknown worm, we can only rely on anomaly detection methods to detect whether a host is infected or not. Anomaly detection methods will always generate false alarms once in a while. If the false alarm rate is high and we release the quarantine on an alarmed host only after manual inspection by security staffs, then many healthy hosts will be quarantined for a long time without normal Internet connections. Such quarantine will dramatically interfere normal activities, which is why people feel hesitated to implement automatic mitigation.

#### 3.1 Dynamic Quarantine Based on Principle “Assume Guilty before Proven Innocent”

Since Internet worms exhibit the similar spreading behavior as infectious diseases in the real world, we can learn from the experiences of epidemic disease control in the real world. For a highly infectious disease that is not easily diagnosed, such as recent SARS disease, people will take aggressive quarantine actions — whenever a person exhibits a
symptom slightly similar to the disease’s, he or she will be quarantined immediately. The quarantine will be released after the person passes the disease latent period without showing up further symptoms of the disease. If the disease is more infectious or the epidemic scale is more severe, the quarantine actions will be more aggressive. Such quarantine will greatly affect the normal life of many healthy people and cost a lot money to our society, but it is the only effective way to deal with a dangerous disease that cannot be diagnosed easily at the disease’s early stage. In other words, in epidemic disease control in the real world, people react under the principle — *assume guilty before proven innocent*.

In this paper, we present a soft dynamic quarantine method based on the same principle: every host of the system can be quarantined individually when the worm anomaly detection program raises alarm for this host; the quarantine on an alarmed host is released after a quarantine time \( T \), even if the host has not been inspected by security staffs yet. Once the quarantine on a host is released, this host can be quarantined again if the anomaly detection program raises alarm for this host again some time later.

If the worm anomaly detection program in the dynamic quarantine system can determine which service port has suspicious activities, then the *quarantine* means we only block traffic on this suspicious port without interrupting normal connections on other ports.

In the real implementation, security staffs should inspect quarantined hosts as quickly as possible. But for fast spreading worms, due to the slow human’s manual response and limited human resources, the inspection by security staffs cannot catch up with the increasing speed of the number of alarmed hosts. Therefore, in order not to severely interfere normal activities, the quarantine on a host will be released automatically after a while even if the host has not been inspected by security staffs yet.

This dynamic quarantine method has two advantages: first, a falsly quarantined healthy host will only be quarantined for a short time, thus its normal activities will not be interfered too much; second, because now we can tolerate higher false alarm rate than the normal permanent quarantine, we can set the worm anomaly detection program to be more sensitive to a worm’s activities. Thus we can detect and quarantine more infected hosts and detect them earlier. The dynamic quarantine method is more effective when we face an unknown stealthy propagating worm that can only be detected with high false alarm rate.

### 3.2 Dynamic Quarantine Analysis

As a first step in this research direction, we study a simple case of dynamic quarantine in this paper: the quarantine time and the anomaly detection threshold are constants throughout the spreading period of a worm.

Suppose on average, an infectious host will be detected in \( 1/\lambda_1 \) units of time after the host becomes infectious, or after it is released from previous quarantine. In other words, an infectious host will propagate on average for about \( 1/\lambda_1 \) time before it is caught and quarantined. We call \( \lambda_1 \) as the *quarantine rate of infectious hosts*.

Any worm anomaly detection program will raise false alarms for healthy hosts from time to time. Suppose on average, a healthy, non-quarantined host will be falsely alarmed by the detection program in the quarantine system once in \( 1/\lambda_2 \) units of time. In other words, a healthy, non-quarantined host will keep its normal activities for \( 1/\lambda_2 \) units of time on average before it is falsely alarmed and quarantined. We call \( \lambda_2 \) as the *quarantine rate of susceptible hosts*. \( \lambda_2 \) corresponds to the false alarm rate of the anomaly detection program used in the system — \( \lambda_2 \) becomes larger if the anomaly detection program has higher false alarm rate.

The values of \( \lambda_1 \) and \( \lambda_2 \) are determined both by the threshold and by the performance of the anomaly detection program used in the system. \( \lambda_1 \) and \( \lambda_2 \) will become larger if the anomaly detection program is set to be more sensitive to a worm’s activities. A high performance anomaly detection program has higher detection rate and lower false alarm rate, i.e., the detection program has larger \( \lambda_1 \) and smaller \( \lambda_2 \) than a worm detection program with ordinary performance.

Denote \( T \) as the quarantine time; \( R(t) \) the number of infectious hosts that are quarantined at time \( t \); \( Q(t) \) the number of susceptible hosts that are quarantined at time \( t \). Let us first derive the formula of \( R(t) \). At time \( t \), all hosts in \( R(t) \) are infectious hosts that are quarantined during time \( (t - T) \) to \( t \) — the hosts that are quarantined before \( (t - T) \) have already been released from the \( R(t) \). At any time \( \tau \), there are \( I(\tau) - R(\tau) \) infectious hosts that are not quarantined. If a quarantined infectious host will not be removed from \( R(t) \) except when its quarantine time is expired, we can derive the formula of \( R(t) \) as

\[
R(t) = \int_{1-T}^{t} [I(\tau) - R(\tau)] \lambda_1 d\tau \tag{5}
\]

Note that Equation (5) is correct only for a large population system because we use the average value \( \lambda_1 \) in it. Each infected host has a variable spreading time before it is quarantined; the variable spreading time has the mean value of \( 1/\lambda_1 \). If \( I(t) - R(t) \) is large, according to the law of large number and from the whole system’s point of view, there will be approximately \([I(\tau) - R(\tau)] \lambda_1 d\tau\) infected hosts are quarantined during the small time interval \( d\tau \).

We cannot, however, derive any strict analytical results from (5) directly — \( R(t) \) depends on previous value of \( I(\tau) \) \( \forall \tau \in [t - T, t] \) and \( I(t) \) will not follow traditional epidemic models (1) or (3) anymore.

In our dynamic quarantine method, the quarantine time \( T \) is small in order not to interfere too much on the normal activities of quarantined healthy hosts. If during the time interval \( T \), \( R(t) \) and \( I(t) \) do not change much, then we can approximately treat them as constants during the time interval \( T \) as

\[
\begin{cases}
R(\tau) \simeq R(t) \\
I(\tau) \simeq I(t)
\end{cases} \quad \forall \tau \in [t - T, t]. \tag{6}
\]

From (5) and (6), we can derive

\[
R(t) = [I(t) - R(t)] \lambda_1 T, \tag{7}
\]

which means we can derive the relationship between \( R(t) \) and \( I(t) \) as

\[
R(t) = p'_1 I(t) \tag{8}
\]

where

\[
p'_1 = \frac{\lambda_1 T}{1 + \lambda_1 T}. \tag{9}
\]
We call $p_1'$ the effective quarantine probability of infectious hosts. Using the same procedure and assumption as (6) by replacing $R(t)$ and $I(t)$ to $Q(t)$ and $S(t)$ respectively, we can derive

$$Q(t) = p_2'S(t)$$  \hspace{1cm} (10)

where

$$p_2' = \frac{\lambda_2T}{1 + \lambda_2T}$$  \hspace{1cm} (11)

is the effective quarantine probability of susceptible hosts.

The analysis above is a general analysis: first, it does not require a specific dynamic model for $I(t)$ and $S(t)$; second, it does not require a specific distribution of the detection time $1/\lambda_1$ and $1/\lambda_2$. The analysis relies on the assumption that the changing speed of $R(t), I(t)$ and $S(t)$ during the time interval $T$ is small. In addition, in order to derive (5), we assume that a quarantined host will not be removed from $R(t)$ or $Q(t)$ unless its quarantine time reaches $T$ (we will show how to relax this requirement in the next section).

In the next section, we will study how the dynamic quarantine affects a worm’s propagation by extending the simple epidemic model (1) and the Kermack-McKendrick model (3), respectively.

4. WORM PROPAGATION MODELING UNDER DYNAMIC QUARANTINE

4.1 Worm Modeling Based on Simple Epidemic Model

We first analyze the impact of dynamic quarantine on a worm’s propagation based on the simple epidemic model (1). As in the simple epidemic model, we assume the system is a homogeneous system with $N$ hosts. No host will be removed from the system — a host is either susceptible or infectious. Due to the dynamic quarantine, a host is either quarantined or not quarantined at any time $t$.

![Figure 1: Interactions between infectious and susceptible hosts](image)

The simple epidemic model (1) is derived based on the interactions between infectious hosts and susceptible hosts. Before we implement dynamic quarantine, a worm propagates according to simple epidemic model (1) with the pairwise rate of infection $\beta$. When we implement dynamic quarantine on the system, Fig. 1 shows that the interactions now are between $[I(t) - R(t)]$ and $[S(t) - Q(t)]$. Therefore, a worm’s propagation under dynamic quarantine follows

$$\frac{dI(t)}{dt} = \beta[I(t) - R(t)][S(t) - Q(t)] = \beta'I(t)[N - I(t)]$$  \hspace{1cm} (12)

where

$$\beta' = (1 - p_1')(1 - p_2')\beta$$  \hspace{1cm} (13)

is the effective pairwise rate of infection.

Equation (12) shows that under dynamic quarantine, a worm still propagates according to simple epidemic model but with slower spreading speed. The dynamic quarantine decreases a worm’s pairwise rate of infection $\beta$ by the factor of $(1 - p_1')(1 - p_2')$: the larger the effective quarantine probabilities $p_1'$ and $p_2'$ are, the slower the worm can propagate. Therefore, when we implement the dynamic quarantine, it can provide us precious time to take counteractions — patching vulnerable computers and cleaning infected ones — before a worm infects the major part of a network.

4.2 Worm Modeling Based on Kermack-McKendrick Epidemic Model

Next, we analyze the impact of dynamic quarantine on a worm’s propagation based on the Kermack-McKendrick epidemic model, i.e., we consider the removal process of infectious hosts. As in the Kermack-McKendrick epidemic model, $U(t)$ is the number of removed hosts from infectious and it follows $dU(t)/dt = \gamma I(t)$ as shown in (3). For the dynamic quarantine system, we assume that we remove infectious hosts uniformly from $I(t)$, regardless of whether a removed host is under quarantine or not when we remove it.

Before we consider removal process, $R(t) = p_1'I(t)$ and $Q(t) = p_2'S(t)$. When we consider removal process of infectious hosts, since it has nothing to do with susceptible hosts, $Q(t) = p_2'S(t)$ still holds. However, Equation (5) should be modified to consider the removed hosts from $R(t)$ during the time $(t - T)$ to $t$. Since the removal process uniformly removes infectious hosts from $I(t)$, the removal rate from quarantined $R(t)$ should be $\gamma R(t)$ at time $t$. Therefore, we can extend (5) to derive

$$R(t) = \int_{t-T}^{t} [I(\tau) - R(\tau)]\lambda_1 d\tau - \int_{t-T}^{t} \gamma R(\tau) d\tau.$$

With the same assumption that $R(\tau) \approx R(t)$ and $I(\tau) \approx I(t)$, $\forall \tau \in [t - T, t]$, from (14) we can derive

$$R(t) = q_1'I(t),$$  \hspace{1cm} (15)

where

$$q_1' = \frac{\lambda_1 T}{1 + (\lambda_1 + \gamma)T}$$  \hspace{1cm} (16)

is the effective quarantine probability of infectious hosts for a worm’s propagation with removal process. For consistence, we denote

$$q_2' = p_2' = \frac{\lambda_2T}{1 + \lambda_2T}$$  \hspace{1cm} (17)

as the effective quarantine probability of susceptible hosts for a worm’s propagation with removal process, i.e.,

$$Q(t) = q_2'S(t).$$  \hspace{1cm} (18)

A worm’s propagation follows

$$\frac{dI(t)}{dt} = \beta[I(t) - R(t)][S(t) - Q(t)] - \gamma I(t) = \beta' I(t)[N - I(t)] - \gamma I(t)$$  \hspace{1cm} (19)

where

$$\beta'' = (1 - q_1')(1 - q_2')\beta$$  \hspace{1cm} (20)
is the effective pairwise rate of infection for a worm’s propagation with removal process.

The worm propagation model (19) is the same as the Kermack-Mckendrick model (3), except that the pairwise rate of infection \( \beta'' \) is decreased from \( \beta \) by the factor of \((1 - q'_1)(1 - q'_2)\). The new dynamic quarantine system will have an epidemic threshold \( \rho' \) that is

\[
\rho' \equiv \frac{\gamma}{\beta''} = \frac{1}{(1 - q'_1)(1 - q'_2)} \rho. \tag{21}
\]

\( \rho' \) is increased from the original value \( \rho \) by the factor of \((1 - q'_1)(1 - q'_2)\). If the initial number of susceptible hosts \( S(0) \) has the relationship \( S(0) > \rho \) and \( S(0) < \rho' \), then according to the Kermack-Mckendrick epidemic threshold theorem, a worm will spread out in the original system but will not be able to spread out when we implement dynamic quarantine on the system. In other words, the dynamic quarantine method reduces the chance for a worm to form an outbreak.

### 4.3 Worm Modeling by Considering the Cleaning of Quarantined Infectious Hosts

In the previous model (19), all infectious hosts have an equal probability to be removed. However, a more realistic scenario is that security staffs only inspect the hosts that have raised alarm and have been quarantined. The reasons are: first, the limited human resources do not permit the full-scale inspection of all hosts; second, alarmed hosts are more likely to be infected by a worm. Therefore, in such a dynamic quarantine system, only infectious hosts in the quarantined population \( R(t) \) could be removed.

In this case, the number of removed hosts \( U(t) \) (from quarantined infectious hosts \( R(t) \)) follows \( dU(t)/dt = \gamma R(t) \). The formula (14) is still correct for this situation. Now the worm propagation model is

\[
d I(t)/dt = \beta I(t) R(t)[S(t) - Q(t)] - \gamma R(t) \tag{22}
\]

where

\[
\gamma' = q_1 \gamma \tag{23}
\]

is the effective removal rate for this system.

We can see that the model (22) has the same format as the Kermack-Mckendrick model (3). Therefore, all theorems of the Kermack-Mckendrick model are valid here. Define the epidemic threshold \( \rho'' \) as

\[
\rho'' \equiv \gamma' / \beta'' = \frac{q'_1}{(1 - q'_1)(1 - q'_2)} \frac{\gamma}{\beta} \tag{24}
\]

The epidemic threshold theorem states that if \( S(0) < \rho'' \), a worm will not form an outbreak in this dynamic quarantine system.

Note that all our analysis formulas are based on two assumptions: first, the quarantine time \( T \) is small such that

\[
\left\{ \begin{array}{ll}
R(\tau) \simeq R(t) & \forall \tau \in [t - T, t]; \\
I(\tau) \simeq I(t); & \forall \tau \in [t - T, t]; \\
S(\tau) \simeq S(t)
\end{array} \right. \tag{25}
\]

second, Equation (5) and (14) rely on the law of large number since these two equations use the mean values of \( \lambda_1 \) and \( \lambda_2 \) without considering stochastic effects — these two equations are accurate only when \( I(t) - R(t) \) is large (the formula of \( Q(t) \) is correct only when \( S(t) - Q(t) \) is large). In the next section, we will use simulations to demonstrate how these two assumptions affect the accuracy of our analysis.

## 5. SIMULATION EXPERIMENTS

### 5.1 Simulation Settings

Worm propagation is a discrete-event dynamic system; event-driven simulation is the most accurate method to simulate the propagation of a worm. However, we are interested in the propagation of a worm in a large network system — an event-driven simulation will be too time-consuming. Therefore, we use discrete-time simulation in this paper.

We try to simulate a worm similar to the Slammer worm on January 25th, 2003. According to [12], Slammer sent out on average 4,000 scans per host per second at the worm’s early growth phase. From their monitors, the authors in [12] observed about 75,000 infected hosts in the first 30 minutes. Therefore, in our simulation, we assume that the vulnerable population is \( N = 75,000 \) and the worm’s average scan rate is \( \eta = 4000 \) per second. The authors in [19] provide a formula to estimate the size of vulnerable population from a worm’s scan rate and infection rate. We can reversely use that formula to derive the worm’s infection rate \( \alpha = nN/2^{32} = 0.0698 \), i.e., an infected host can probe on average 0.0698 hosts among those 75,000 initially vulnerable ones. We also assume \( I(0) = 10 \), i.e., 10 vulnerable hosts in the system are infectious at the beginning.

To increase the accuracy of our discrete-time simulation, we use 0.05 second as the discrete time unit, i.e., the simulation program will iterate 20 times for simulating 1 second of a worm’s propagation.

When we consider the dynamic quarantine, we assume that the time before a host is alarmed follows exponential distribution: the quarantine rate of infectious hosts is \( \lambda_1 = 0.2 \) per second, i.e., on average an infectious host can propagate for about 5 seconds before it is alarmed and quarantined; the quarantine rate of susceptible hosts is \( \lambda_2 = 0.000002315 \) per second, i.e., the worm anomaly detection program will give on average twice false alarms for a healthy host per day. We set the quarantine time to be \( T = 10 \) seconds.

### 5.2 Worm Propagation without Considering Removal Process

We first consider a worm’s propagation without removal of infectious hosts. In this case, in the original system where there is no dynamic quarantine, a worm will propagate according to the simple epidemic model (1). Fig. 2(a) shows the number of infectious hosts \( I(t) \) as a function of time \( t \) when a worm propagates in the dynamic quarantine system. It compares the worm’s propagation in the dynamic quarantine system with the worm’s propagation in the original system. This figure shows that in the dynamic quarantine system, a worm still propagates according to the epidemic model (1), but propagates at a much slower speed.

Fig. 2(b) shows the dynamics of \( I(t) \), \( R(t) \) and \( Q(t) \) as functions of time \( t \). Because \( \lambda_2 \) is very small, the number of quarantined susceptible hosts, \( Q(t) \), is much smaller than \( I(t) \) and \( R(t) \). Thus we enlarge \( Q(t) \) by 500 times in order to show \( I(t) \), \( R(t) \), and \( Q(t) \) in the same figure. This figure
shows that the random effect of a worm’s propagation shows up in the small value of $Q(t)$; but because of the law of large number, the curves of $I(t)$ and $R(t)$ are smooth.

In order to verify the formulas of $R(t)$ and $Q(t)$ in (8) and (10), we calculate the ratio of $R(t)/I(t)$ and $Q(t)/S(t)$ from the simulation at each second $t = 1, 2, \cdots$. We plot these two ratios as functions of time $t$ in Fig. 2(c) compared with their theoretical values from (9) and (11). Because the value of $p_2'$ is very small, we enlarge it 500 times in order to show $p_1'$ and $p_2'$ in the same figure. This figure shows that the formulas (9) and (11) are accurate for most part of a worm’s propagation. Even when the assumption (25) is not accurate during the worm’s fast spreading period (from 250 seconds to 400 seconds when $R(t)$ and $I(t)$ increase quickly), the formulas (9) and (11) still hold.

Fig. 2(c) shows that the formula (9) of $p_1'$ is not accurate at the beginning of a worm’s propagation. This is because the formula (9) relies on the law of large number: at the beginning when $I(t)$ is small, it is not accurate to directly use the mean value $\lambda_1$ to calculate $p_1'$. This is also the reason of the large oscillation of $p_2'$ at the end of a worm’s propagation when $S(t)$ is small. In the whole process of a worm’s propagation, the large oscillation of $p_2'$ is due to the small and variable $Q(t)$ as shown in Fig. 2(b).

### 5.2.1 Variability in Worm Propagation

Worm propagation is in fact a stochastic process. A small random variations at the beginning of a worm’s growth can affect dramatically how quickly the worm spreads [11]. Here we conduct experiments to check how the variability in a worm’s propagation affects our analysis. With the same simulation parameters above, we run the simulations for 100 times. Fig. 3(a) shows the upper and lower bounds and the average value of the number of infectious hosts in these 100 simulation runs.

For each of these 100 simulation runs, we calculate the ratio $p_1' = R(t)/I(t)$ after the worm infects 1% of the population (the worm in different simulation runs will take different lengths of time to infect 1% hosts). Then we obtain the maximum and minimum values of the observed $p_1'$ for each simulation run — the oscillation of the observed $p_1'$ will not exceed this boundary after the worm infects 1% population. We plot this boundary in Fig. 3(b) for each of these 100 simulation runs. In order to check if the formula of $p_1'$ becomes less accurate when a worm propagates faster, in Fig. 3(b) we have sorted these 100 simulation runs according to the time when the worm infects 1% population. In other words, the worm in simulation $i$ infects 1% of vulnerable hosts earlier than the worm in simulation $j$ does if $i < j$. This figure shows that the accuracy of our analysis does not depend on how a worm propagates in different situations — if a worm propagates faster, i.e., $I(t)$ increases faster, then the number of quarantined infectious hosts $R(t)$ will also increase faster accordingly.

### 5.2.2 Effect of a Large Quarantine Time $T$

The simulation in Fig. 2 shows that our analysis is robust to the assumption in (25). Then what happens if we select a larger quarantine time $T$? To answer this, we run another simulation with $T = 30$ seconds and show the simulation results in Fig. 4. In this simulation, we try to let a worm to propagate in the similar speed as the one shown in Fig. 2; thus we choose $\lambda_1 = 0.2/3$ and $\lambda_2 = 0.00002315/3$ in order to let $p_1'$ and $p_2'$ in this simulation to have the same values as in the simulation in Fig. 2. All other parameters are the same as what used in that simulation.

According to our analysis, in this simulation a worm should propagate with the same speed as the one shown in Fig. 2(b). However, Fig. 4(a) shows that in this simulation, the worm propagates a little bit faster. This is because the assumption (25) in our analysis is not accurate anymore for this simulation. During the fast increasing part of $I(t)$ and $R(t)$ (before time 350 seconds), $I(t)$ and $R(t)$ will have

$$
\begin{cases}
R(\tau) < R(t) \\
I(\tau) < I(t)
\end{cases} \quad \forall \tau \in [t - T, t];
$$

(26)

thus (7) will become $R(t) < [I(t) - R(t)]\lambda_1 T$. In this case, the relationship between $R(t)$ and $I(t)$ is

$$R(t) < p_1' I(t)
$$

(27)

instead of the formula (8). Fig. 4(b) verifies this analysis — the observed $p_1'$ is smaller than the theoretical value from (9) before time 350. Since the number of quarantined infectious hosts is smaller than the one in the simulation shown in
5.3 Worm Propagation Considering Removal Process

5.3.1 Quarantine System Described by Model (19)

Now we study a worm’s propagation when we consider the removal of infectious hosts. First, we consider the dynamic quarantine system described by the model (19), i.e., all infectious hosts have the equal probability to be removed regardless whether they are quarantined or not.

We briefly explain how we choose the removal rate $\gamma$. To study a worm’s propagation, we need to let the worm to spread out, which means we should select parameters such that $S(0) > \rho'$ according to the epidemic threshold theorem. Since $S(0) = N - I(0) \approx N$, from (2), (4), and (21), we should select $\gamma$ to satisfy

$$\gamma < (1 - q_1')(1 - q_2')\alpha < \alpha$$

Thus $\alpha$ is an upper bound for $\gamma$. From (16), we know that

$q_1' > \frac{\lambda_1 T}{1 + (\lambda_1 + \alpha)T}$. Thus a tighter upper bound for $\gamma$ is

$$\gamma < (1 - \frac{\lambda_1 T}{1 + (\lambda_1 + \alpha)T})(1 - q_2')\alpha = \frac{(1 + \alpha T)\alpha}{(1 + \lambda_2 T)(1 + (\lambda_1 + \alpha)T)}$$

In this simulation, we use the same parameters as what used in the simulation shown in Fig. 2. In this case, Equation (29) shows that we should choose $\gamma < 0.032$. Therefore, we choose $\gamma = 0.01$ in the simulation.

The simulation results are shown in Fig. 5; this figure has the same format and meanings as Fig. 2. The “original system” in Fig. 5(a) is the non-quarantine system described by the Kermack-Mckendrick model (3). Note that the Y-axis scales in Fig. 5(a)(b) are different. Fig. 5 shows that our analysis and the model (19) are correct: in a dynamic quarantine system with the removal process, a worm propagates according to the model (19) with much slower propagation speed than the worm does in the original system without dynamic quarantine defense.

5.3.2 Quarantine System Described by Model (22)

Next we consider the dynamic quarantine system described by the model (22), i.e., only quarantined infectious hosts are
possible to be removed. The simulation results are shown in Fig. 6, which has the same format and meanings as Fig. 5. The "original system" in Fig. 6(a) is the non-quarantine system without removal process, i.e., a worm’s propagation in this system can be described by the simple epidemic model (1). Fig. 6 shows that our analysis and the model (22) are correct; in such a dynamic quarantine system, a worm propagates much slower and follows the model (22).

6. CONCLUSION

Enlightened by the methods used in epidemic disease control in the real world, we present a dynamic quarantine method based on the principle “assume guilty before proven innocent”. We quarantine a host whenever its behavior looks suspicious by blocking traffic on the anomaly port, then we will release the quarantine after a short time, even if the host has not been inspected by security staffs yet. As a first step, in this paper we analyze the dynamic quarantine system that has constant quarantine time and worm detection threshold. Our mathematical analysis shows that in the dynamic quarantine system, a worm still propagates according to traditional epidemic models, but with slower propagation speed and higher epidemic threshold.

To derive simple mathematical formulas, in this paper we have simplified the quarantine system and the dynamics of a worm’s propagation. For example, we have assumed that all hosts in the system have the same quarantine rates \( \lambda_1 \) and \( \lambda_2 \). We need to further study the case where each host has different quarantine rates. In order to use classical epidemic models, we also have assumed that the system is homogeneous and the contact rate is constant for all hosts at any time. We need to study how to extend the analysis in this paper to a non-homogeneous system with variable contact rate.

A more advanced dynamic quarantine system should have dynamically changing quarantine time and detection threshold during a worm’s propagation. Like what people act in epidemic disease control in the real world, if a worm is more infectious and poses more damage to our networks, the dynamic quarantine defense should be more aggressive — the anomaly detection should be more sensitive to the worm’s activities, and the quarantine time should become longer to further constrain quarantined infectious hosts. Our long-term objective is to develop a “feedback control dynamic quarantine system”. This feedback quarantine system can optimally adjust the anomaly detection threshold and the quarantine time in order to minimize the cost of false alarms.
and at the same time to slow down a worm’s spreading speed as much as possible. This paper is our first step into that direction.

7. ACKNOWLEDGEMENTS

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8. REFERENCES