# Evolutionary Non-Cooperative Spectrum Sharing Game: Long Term Coexistence for Collocated Cognitive Radio Networks

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**Abstract**

Collocated cognitive radio networks (CRNs) employ coexistence protocols to share the spectrum when it is not being used by the licensed primary users. These protocols work under the assumption that all spectrum bands provide the same level of QoS which is somewhat simplistic because channel conditions as well as the licensee's usage of allocated channels can vary significantly with time and space. These circumstances dictate that some channels may be considered *better* than others, therefore CRNs are expected to have a preference over the choice of available channels. Since all CRNs are assumed to be rational and select the best available channels, it can lead to an imbalance in contention for disparate channels, degraded QoS and an overall inefficient utilization of spectrum resource. In this paper, we analyze this situation from a game theoretic perspective and model the coexistence of CRNs with heterogeneous spectrum as an evolutionary anti-coordination spectrum sharing game. We derive the evolutionarily stable strategy (ESS) of the game by proving that it cannot be invaded by a greedy strategy. We also derive the *Replicator Dynamics* of the proposed evolutionary game, a mechanism with which players can learn from their payoff outcomes of strategic interactions and modify their strategies at every stage of the game and subsequently converge to ESS. Since all CRNs approach ESS based solely upon the common knowledge payoff observations, the evolutionary game can be implemented in a distributed manner. Finally, we analyze the game from the perspective of fairness using Jain's fairness index under selfish behavior from CRNs.

**Index Terms**

Cognitive Radio Networks, Coexistence, Evolutionary Game Theory, Evolutionarily Stable Strategy.

## I. Introduction

The Federal Communications Commission (FCC) made TV white space (TVWS) channels in the 54-698 MHz frequency range available [2] for secondary unlicensed access after the TV broadcast was switched from analog to digital signal in 2009. Opening up of the TVWS for unlicensed use was the result of

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a realization that the gap between the demand and supply of wireless spectrum resource is ever increasing and fixed spectrum allocation is causing its severe under-utilization [3]. Strict requirements are however placed on the Secondary Users (SUs) of the spectrum which is otherwise allocated to licensees called primary users (PU), to continuously sense the spectrum and vacate it when the presence of the PU is detected and not to cause them any interference. This type of spectrum access is intuitively called Dynamic Spectrum Access (DSA). Cognitive Radio Network (CRN) is a paradigm that meets precisely these communication requirements and utilizes DSA to enable secondary, unlicensed access to TVWS spectrum bands in an opportunistic and non-interfering basis [2].

DSA allows CRNs to ensure that their use of spectrum does not cause interference to PUs while at the same time all spectrum opportunities are utilized to the maximum. Within a CRN, the decision to select a specific channel for DSA is usually made by a central entity such as its base station or in case of an ad hoc CRN, an algorithm that enables all SUs to reach a consensus for choosing specific channel in a distributed manner. IEEE 802.22 wireless regional area network (WRAN) [4] is an example of CRNs with very large transmission ranges (from 30 to 100 kms) in which the base station controls all the operation of the CRN including the choice of spectrum bands for communication. Regardless of how a decision to select a specific channel is made, every entity within the CRN is bound to abide by that decision. On the other hand, there may be multiple collocated CRNs within a geographical region all of which compete for access to the same set of available channels. Sharing of spectrum by collocated CRNs is called self coexistence in the context of CRNs which employ coexistence protocols such as the IEEE 802.22 standard's Coexistence Beacon Protocol (CBP). However without any controlling entity, fair distribution of *heterogeneous* spectrum resources is non-trivial in the case of multiple collocated CRNs as they may be independently owned and operated by different service providers. This brings us to the definition of this paper's problem statement for long term coexistence with heterogeneous spectrum, in the following subsection.

### A. Problem Definition

Coexistence protocols employed by collocated CRNs work under the assumption that all spectrum bands afford the same level of QoS and do not take into consideration the fact that these channels can be heterogeneous. The heterogeneity of channels can be in the sense that they may vary in their characteristics such as Signal-to-Noise ratio (SNR) or bandwidth. Similarly, a channel whose PU remains idle for most of the time may be more attractive for a CRN as compared with a channel whose PU remains mostly active. This would entail that some channels can be considered better than others and therefore can have an associated *quality* parameter. As a result, CRNs are expected to have a preference over the set of available channels for secondary access. Without any incentive for altruism, all CRNs would want to gain access to the highest quality channels resulting in a conflict among rational entities. Therefore, in the absence of any centralized enforcement mechansim, *evolution of a strategy that would*

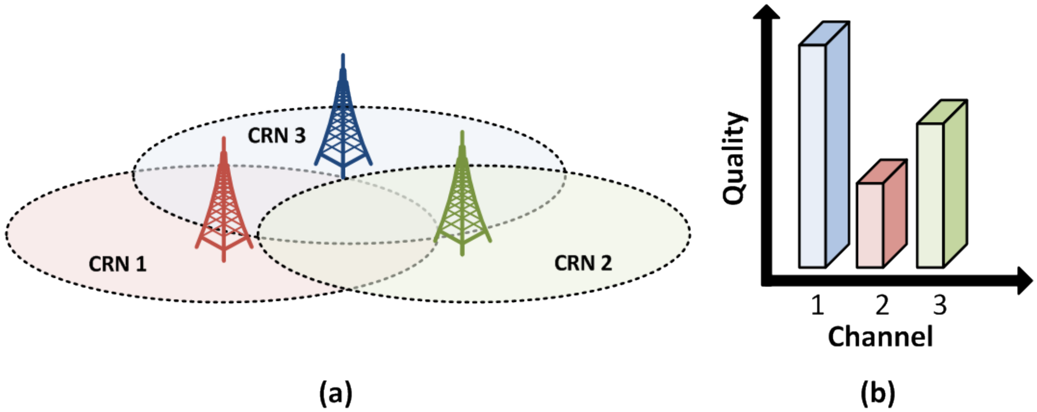


Fig. 1: (a) Collocated CRNs competing for (b) Heterogeneous channels. The channels of the spectrum band may vary in quality with respect to availability, bandwidth or SNR, etc.

*ensure long term coexistence with fair distribution of heterogeneous spectrum resources* among collocated CRNs is a challenge and is the problem statement for this paper.

### *B. Game Theoretic Approach*

Game theory provides an elegant means to model strategic interaction between agents which may or may not be cooperative in nature. It has been applied to numerous areas of research involving conflict, competition and cooperation in multi-agent systems which also encompass wireless communications. Therefore, by leveraging the mechanisms of game theory, we model the long term sharing of heterogeneous spectrum by CRNs as an evolutionary anti-coordination spectrum sharing game in which collocated CRNs in a given region are its players. The payoff for every player in the game is determined by the quality of the spectrum band to which it is able to gain access.

In this paper, we present a detailed analysis on the evolutionary stability as well as fairness of the solution. For any system with non-cooperative entities, it is likely that there will be some associated inefficiency. However, it is worth pointing out that *fairness* is the primary objective of our proposed evolutionary heterogeneous spectrum sharing game. We also confirm our findings through detailed simulations.

### *C. Contribution*

In this paper, we have formulated an evolutionary spectrum sharing anti-coordination game and propose its solution that is stable even with the presence of greedy strategy, robust under changing network conditions and at the same time results in fair distribution of the spectrum resources. Specifically, we have made the following contributions:

- As potential solutions for the heterogeneous spectrum sharing game, we have derived the game's pure and mixed strategy Nash Equilibria (PSNE and MSNE respectively).

- To show that the game's strategy in MSNE is evolutionarily stable strategy (ESS), we prove that it cannot be invaded by a greedy strategy and is robust under changing network conditions.
- We have derived replicator dynamics of the proposed evolutionary game, a mechanism with which players can learn from their payoff outcomes of strategic interactions and modify their strategies at every stage of the game and subsequently converge to ESS.
- Finally, we have presented a fairness analysis of the proposed evolutionary game using Jain's fairness index.

The rest of this paper is organized as follows: Section II provides on overview of the existing work on various solutions for spectrum access and sharing. Section III presents the underlying system model and assumptions for our proposed evolutionary spectrum sharing game while section IV presents the formulation of our proposed evolutionary game along with its replicator dynamics. Section V presents the fairness analysis of the evolutionary game and simulation results are presented in section VI. Section VII concludes the paper.

## II. Related Work

In this section we provide an overview of some of the works carried out in the domain of self coexistence in CRNs as well as application of some of the game theoretic solution concepts in the context of communication networks.

A spectrum sharing mechanism is proposed in [5] in which the PUs lease their licensed spectrum bands to SUs in return for their cooperation in relaying PUs' traffic. The work proves that the spectrum sharing game converges to a stackelberg equilibrium. Authors of [6] have developed an auction-based algorithms for joint allocation of resources, i.e., source and relay nodes power profiles and subcarrier assignment for amplify and forward OFDMA systems. It is based on a one-shot auction where each user submits bids for all subcarriers at once based on the Shapley value.

Authors of [7] have applied the evolutionary game theoretic concepts in order to make secondary users (SUs) of a CRN to participate in collaborative spectrum sensing in a decentralized manner. SUs learn through strategic interactions at every stage of the game and the learning behavior is modeled with the help of replicator dynamics. A game theoretic approach based on correlated equilibrium has been proposed in [8] for multi-tier decentralized interference mitigation in two-tier cellular systems. Authors of [9] propose a multi-cell resource allocation game for efficient allocation of resources in orthogonal frequency division multiple access (OFDMA) systems based on throughput, inter-cell interference and complexity. The subcarriers are considered as players of the game while the base station acts as the provider of external recommendation signal needed for achieving correlation of strategies of players.

Authors of [10] model the competition among multiple femtocell base stations for spectrum resource allocation in an OFDMA LTE downlink system as a static non-cooperative game. The correlated equilibrium of the game is derived through a distributed resource block access algorithm which is a variant of the No-Regret learning algorithm. CRNs with SUs having variable traffic characteristics are considered in [11] to tackle the problem of distributed spectrum sensing by modeling it as a cooperative spectrum sensing game for utility maximization. The authors have proposed another variant of the no-regret learning algorithm called neighborhood learning (NBL) which achieves correlated equilibrium for the spectrum sensing game. In contrast to the no-regret learning algorithm, NBL is not completely distributed and requires some coordination among players to achieve better performance.

Correlated equilibrium has been employed in [12] for a P2P file sharing non-cooperative game to jointly optimize players expected delays in downloading files. Not uploading files for others causes an increase in file download time for all players which in turn, forces even the non-cooperative players to cooperate. The authors of [13] tackle the self-coexistence problem of finding a mechanism that achieves a minimum number of wasted time slots for every collocated CRN to find an empty spectrum band for communications. To do so, they employ a distributed modified minority game under incomplete information assumption.

Different punishment strategies have been employed in [14] that form part of a Gaussian interference game in a one-shot game as well as an infinite horizon repeated game to enforce cooperation. Spectrum sharing is however considered within the context of a single CRN. Evolutionary game theory is applied in [15] to solve the problem in a joint context of spectrum sensing and sharing within a single CRN. Multiple SUs are assumed to be competing for unlicensed access to a single channel. SUs are considered to have half-duplex devices so they cannot sense and access a channel simultaneously. Correlated equilibrium has been proposed in [16] as a solution for efficient coexistence by collocated CRNs with heterogeneous channels.

Utility graph coloring is used to address the problem of self-coexistence in CRNs in [17]. Allocation of spectrum for multiple overlapping CRNs is done using graph coloring in order to minimize interference and maximize spectrum utilization using a combination of aggregation, fragmentation of channel carriers, broadcast messages and contention resolution. The authors of [18] achieve correlated equilibrium with the help of No-regret learning algorithm to address the problem of network congestion when a number of SUs within a single CRN contend for access to channels using a CSMA type MAC protocol. They model interactions of SUs within the CRN as a prisoners dilemma game in which payoffs for the players are based on aggressive or non-aggressive transmission strategies after gaining access to idle channels.

## III. System Model and Assumptions

### A. System Model

As shown in figure 1, we consider a region where collocated and overlapping CRNs co-exist and compete with each other for secondary access to the licensed spectrum bands. We model the entire TVWS spectrum band that is available for unlicensed use by CRNs as a set of $K = 1, 2, ..., k$ channels. The spectrum band is heterogeneous by virtue of the quality of a channel which is determined by the probability $P_k$ with which PUs access their licensed channels. Since knowledge of PU's spectrum allocation/activity is mandated by the FCC for CRNs [2], [4], is publically available through online databases [19], [20] and also sensed by CRNs at regular intervals, players can calculate current values of $P_k$ based on past observations. Higher $P_k$ for a given channel $k$ means it is of a lower quality and vice versa and CRNs compete to access the best quality channels. Gaining access to higher quality channel results in higher payoff $u_k$ while lower quality channel yields lower payoff for CRNs where payoff $u_k = 1 - P_k$ from gaining access to channel $k$. Since every CRN is required to sense for the presence of PU on the spectrum, all of them can calculate $P_k$ and hence the payoffs of all the channels are considered common knowledge as they would be the same for every player. CRNs need to gain access to a channel in every time slot also called a Channel Detection Time (CDT) slot [4]. Players are assumed to be rational and non-cooperative i.e., they do not share a common goal and therefore do not cooperate with each other. It is in every CRN's interest to gain access to the channels with minimum PU activity i.e., minimum value of $P_k$. When two or more CRNs select the same channel for access in a given time slot, a contention/collision situation arises and that particular time slot's spectrum opportunity is wasted. Having payoffs for selecting a specific channel derived from common knowledge such as $P_k$ is an intuitive choice and makes distributed implementation of our proposed framework possible. It is worth mentioning that any positive value for payoff derived from any other parameter e.g., QoS or bandwidth can be used instead of $P_k$ without affecting our analysis and the outcomes. As demonstrated subsequently, the number of collocated CRNs does not play any part in the game model because an evolutionary game is concerned with the *evolution of strategies*, associated payoffs and their stability.

### B. Assumptions

Following are the underlying assumptions for the work presented in this paper:

- ***Time:*** A single MAC superframe constitutes one time slot. Every CRN needs to gain access to a channel for which it contends with all other collocated CRNs in every time slot.
- ***Spectrum opportunity and wastage:*** A given time slot's spectrum opportunity arises due to the absence of its PU may result in a collision and therefore be wasted if two or more CRNs select the same channel for access.
- ***Knowledge about PU activity:*** In addition to the FCC mandated continuous spectrum sensing to detect PUs' activity, CRNs are also required to periodically access online databases such as [19], [20] in order to gain up-to-date information about licensed PUs operating in a given region.
- ***Channel quality:*** The amount of PU activity, bandwidth and SNR which, for the purpose of this paper collectively determine a channel's quality can be learnt from online databases and measured through spectrum

TABLE I: Notations & Acronyms

| **Notation** | **Definition** |
|---|---|
| $\mathcal{K}$ | set of available channels |
| $\mathcal{A}$ | set of available actions (selecting channels) |
| $\mathcal{U}$ | set of channels' utilities |
| $a_k$ | CRN's action of selecting channel $k$ |
| $u_k$ | CRN's utility for gaining access to channel $k$, $u_k$ may be any positive number |
| $a_i$ | action/strategy played by player $i$ |
| $a^*_{-i}$ | best actions/strategies played by players other than player $i$ |
| $a^*_i$ | action/strategy of player $i$ which is the best response (PSNE) to $a^*_{-i}$ |
| $\hat{p}$ | ESS prob. distr over set of channels (in MSNE) |
| $p'$ | A mutant strategy that is greedier than ESS strategy |
| $EU_k$ | Expected Utility from accessing channel $k$ |
| $t$ | current time |
| CDT | Channel Detection Time (1 time slot) |
| ESS | Evolutionarily Stable Strategy |
| PU | Primary User |
| SU | Secondary User |
| NE | Nash Equilibrium |
| PSNE | Pure Strategy Nash Equilibrium |
| MSNE | Mixed Strategy Nash Equilibrium |

sensing over a period of time. Due to the fact that all contending CRNs are collocated in a given region, it is reasonable to assume that a given channel's quality is common knowledge.

- ***Non-cooperative behavior:*** All CRNs are independent as they do not share a common goal and therefore do not cooperate with each other. Being rational about their choices [24], every player has a clear preference of selecting the best available channel before the start of every time slot. As a consequence of assuming rational behavior from them, players aim to maximize only their own payoffs. Therefore, if every player tries to access the best channel in a given time slot, it will result in a collision and the spectrum opportunity being wasted.
- ***Payoffs*[1]*:*** Players[2] that eventually gain access to higher quality channels will gain higher payoffs as compared to

[1]We use the terms utility and payoff interchangeably.

[2]Similarly, we use the terms CRNs and players interchangeably

the players that end up with lower quality channels. In subsequent section, we show that our proposed spectrum sharing game can be implemented solely on the basis of a CRN's common knowledge payoff observations.

## IV. Evolutionary Anti-Coordination Spectrum Sharing Game

In this section, we first present the basics of evolutionary game theory followed by formulation of our proposed evolutionary spectrum sharing game. Next, we derive solutions for the game for a 2-channel scenario and extend it for a $K$-channel scenario with replicator dynamics.

### *A. Evolutionary Game Theory: Basics*

Evolutionary game theory formalizes the way in which various strategies of a population mix interact while competing against each other. As a result of such competitions, relative *fitness* of strategies can be determined based upon the payoffs that the strategies bring. An incumbent strategy of a population may be invaded by a mutant strategy if, on average, the mutant strategy can bring higher payoffs than the incumbent strategy. A strategy that cannot be invaded by a mutant strategy is said to be an evolutionarily stable strategy or ESS. In this paper, we consider the action of selecting a specific channel as a CRN's strategy and need to determine which strategies are fair and stable for the long term. To that end, we derive the PSNE and MSNE as the game's solutions and prove that MSNE is ESS i.e., MSNE cannot be invaded by a mutant strtaegy that is greedier than MSNE. In addition to being evolutionarily stable, MSNE of the game is also fair because of its definition, which is presented subsequently.

### *B. Game Formulation*

The heterogeneous spectrum sharing anti-coordination game presented in this paper is a non-cooperative repeated game with perfect information because:

- Being rational players, CRNs compete for the best channels available in the spectrum band and are interested only in maximizing their own utility. Therefore, CRNs are not bound to cooperate with each other.
- Utilities are common knowledge since the quality of various network parameters can be measured by every CRN. Also, every CRN can tell which channels other CRNs were able to gain access to in the past hence they know other CRNs' payoffs.

The evolutionary heterogeneous spectrum sharing game is represented as $\mathcal{G} = \langle(\mathcal{K}), (\mathcal{A}), (\mathcal{U})\rangle$ where $\mathcal{K} = \{1, 2, ..., k\}$ denotes the set of available channels. Every player in the game has the same action space represented by $\mathcal{A} = \{a_1, a_2, ..., a_k\}$ and there is a bijection between the sets $\mathcal{A}$ and $\mathcal{K}$. The set of utilities of the channels is represented as $\mathcal{U} = \{u_1, u_2, ..., u_k\}$. Strategy $a_k$ means selecting channel $k$ for communication and a player gets a payoff of $u_k$ if it selected channel $k$ and no other player selected the same channel for a given time slot. The

TABLE II: Strategic form representation of Evolutionary Heterogeneous Spectrum Sharing game with *deterministic* strategies $a_k$ and $a_j$.

| | $a_k$ | $a_j$ |
|---|---|---|
| $a_k$ | $(0,0)$ | $(u_k,u_j)$ |
| $a_j$ | $(u_j,u_k)$ | $(0,0)$ |

payoff for players playing strategies $a_k$ and $a_j$ when competing against each other is denoted by the ordered pair $u(a_k,a_j) \in \mathcal{U}$ and is a function of an individual channel's quality given by:

$$u(a_k,a_j) = \begin{cases} (u_k,u_j) \text{ when } k \neq j \\ (0,0) \text{ when } k = j \end{cases} \qquad u_k,u_j > 0 \tag{1}$$

where the first element of the ordered pair $u(a_k,a_j)$ represents the payoff for player that selected channel $k$ and the second element for player that selected channel $j$. For the sake of clarity and ease in analysis and without any loss of generality, we assume that $u_k > u_j, \forall u \in \mathbb{R}^k_{\geq 0}$. Also, we initially consider a 2-channel game i.e., a game with 2 heterogeneous channels and derive its PSNE and MSNE as potential solutions. Later, we consider the $K$-channel scenario where $K = |\mathcal{K}|$, in section IV-E and derive the *Replicator Dynamics* of the proposed evolutionary game. Replicator dynamics is a mechanism with which players can learn from their payoff outcomes of strategic interactions and modify their strategies at every stage of the game to converge to ESS. The game represented by eq (1) can also be represented in strategic form as table II, which shows the payoffs for two players selecting channels $k$ or $j$. Since $u_k > u_j$, it is in every CRN's interest to choose channel $k$ instead of channel $j$ for a larger payoff. However, when the players select the same channel it results in a collision, the spectrum opportunity being wasted and both player end up with a payoff of $0$. On the other hand, if both players select different channels then their payoffs reflect the quality of the channel to which they are able to gain access, hence the name *anti-coordination* game. As shown in table II, this game is the reverse of the classic *Battle of the Sexes* game and is classified as an anti-coordination game where it is in both players' interest not to end up selecting the same strategy.

### C. Pure and Mixed Strategy Nash Equilibria for the Evolutionary Spectrum Sharing Game

In this subsection we first derive the PSNE followed by MSNE, which are the two potential solutions that are considered for our evolutionary spectrum sharing anti-coordination game.

**Definition 1:** The *Pure Strategy Nash Equilibrium* [21], [24] of the spectrum sharing game is an action profile $a^* \in \mathcal{A}$ of actions, such that:

$$u(a_i^*, a_{-i}^*) \succeq u(a_i, a_{-i}^*), \forall i \in \mathcal{N} \tag{2}$$

where $\succeq$ is a preference relation over payoffs of strategies $a_i^*$ and $a_i$. The above definition means that for $a_i^*$ to be

TABLE III: Strategic form representation of Evolutionary Heterogeneous Spectrum Sharing game with *probabilistic* strategies $a_k$ and $a_j$.

| $\hat{p}$ | $Prob.(a_k) = \alpha$ | $Prob.(a_j) = \beta$ |
|---|---|---|
| $Prob.(a_k) = \alpha$ | $(0, 0)$ | $(u_k, u_j)$ |
| $Prob.(a_j) = \beta$ | $(u_j, u_k)$ | $(0, 0)$ |

a pure strategy NE, it must satisfy the condition that no player $i$ has another strategy that yields a higher payoff than the one for playing $a_i^*$ given that every other player plays their equilibrium strategy $a_{-i}^*$.

***Lemma 1:*** Strategy pairs $(a_k, a_j)$ and $(a_j, a_k)$ are pure strategy NE of the anti-coordination game of table II.

***Proof:*** Assume player 1 to be the row player and player 2 to be the column player in table II. From equation (1) it follows that both $u_k$ and $u_j$ are positive values and therefore the payoffs for strategy pairs $(a_k, a_j)$ and $(a_j, a_k)$ are greater than the payoffs for strategy pairs $(a_k, a_k)$ and $(a_j, a_j)$. Consider the payoff for strategy pair $(a_k, a_j)$ from table II. Given that the player playing strategy $a_j$ continues to play this strategy, then from definition 1 for PSNE, it follows that the player playing strategy $a_k$ does not have any incentive to change its choice to $a_j$ i.e., it will receive a smaller payoff of 0 if it unilaterally switched to $a_j$. Therefore, $(a_k, a_j)$ is a PSNE. The same argument can be applied to prove that the strategy pair $(a_j, a_k)$ is the second PSNE of this game. ■

**Definition 2:** The *Mixed Strategy Nash Equilibrium* [21], [24] of the spectrum sharing game is a probability distribution $\hat{p}$ over the set of actions $A$ for any player such that:

$$\hat{p} = (p_1, p_2, ..., p_{|\mathcal{K}|}) \in \mathbb{R}^{|\mathcal{K}|}_{\geq 0}, \text{and} \sum_{j=1}^{|\mathcal{K}|} p_j = 1 \tag{3}$$

which makes the opponents indifferent about the choice of their strategies by making the payoffs from all of their strategies equal.

Let $\alpha$ be the probability with which player 1 plays strategy $a_k$ and $\beta = (1 - \alpha)$ be the probability of playing strategy $a_j$, then from the payoffs of table III, the expected utility $EU_2(a_k)$ of player 2 for playing strategy $a_k$ is given by:

$$EU_2(a_k) = \alpha u(a_k, a_k) + \beta u(a_j, a_k) = \alpha(0) + \beta u_k \tag{4}$$

Similarly, the expected utility $EU_2(a_j)$ of player 2 for playing strategy $a_j$ is given by:

$$EU_2(a_j) = \alpha u(a_k, a_j) + \beta u(a_j, a_j) = \alpha u_j + \beta(0) \tag{5}$$

According to definition 2, player 2 will be indifferent about the choice of strategies when the expected utilities from playing strategies $a_k$ and $a_j$ are equal, i.e.,

$$EU_2(a_k) = EU_2(a_j) \tag{6}$$

Substituting (4) and (5) in (6), we have $\beta(u_k) = \alpha(u_j)$. Therefore:

$$\alpha = \frac{u_k}{u_k + u_j} \tag{7}$$

$$\beta = 1 - \alpha = \frac{u_j}{u_k + u_j} \tag{8}$$

The mixed strategy NE for the heterogeneous spectrum sharing game is given by the distribution $\hat{p} = \{\alpha, \beta\}$ of equations (7) and (8) which means that when both players select strategies $a_k$ and $a_j$ with probabilities $\alpha$ and $\beta$ respectively, then their opponents will be indifferent about the outcomes of the play. This means that all CRNs in a given region form a polymorphic population in which every CRN mixes for its choice of available channels according to the probability distribution $\hat{p}$ which is the MSNE for our evolutionary channel sharing game. The probability distribution $\hat{p}$ also represents the proportions of the population adopting different strategies at any given stage of the game. To generalize, expected utility for every player $i$ in a $K$-channel heterogeneous spectrum sharing game is given as follows:

$$EU_m = \sum_{m=1}^{|\mathcal{K}|} u_m.p_m, \forall i, m \in \mathcal{K} \tag{9}$$

where $p_m$ represents the probability of a CRN selecting channel $m$ and all other CRNs not selecting channel $m$.

### D. Evolutionary Stability of the Game's Equilibria

To determine if the game's solutions derived in preceding subsection can be invaded by a mutant strategy that is greedier, we analyze its evolutionary stability with the help of definition 3 as follows:

**Definition 3:** For a strategy  to be ESS, it must satisfy the following conditions [22]:

1. $u(\hat{p}, \hat{p}) \geq u(p', \hat{p})$ and
2. if $u(\hat{p}, \hat{p}) = u(p', \hat{p})$ then $u(\hat{p}, p') > u(p', p')$

where $\hat{p}$ is the strategy played by the population and can therefore be termed as the population's incumbent strategy while $p'$ is a mutant strategy that competes with the incumbent strategy. According to the first condition of definition 3, an incumbent strategy (1) must be a symmetric NE and (2) must perform at least as good against itself as it does against a mutant strategy. According to the second condition of definition 3, if an incumbent strategy is not a strict NE then the incumbent strategy must do strictly better against a mutant than a mutant strategy does against itself. Now we analyze both PSNE and MSNE derived in preceding subsection according to definition 3 to see if they are evolutionarily stable.

*1) **Evolutionary Stability of PSNE:*** Earlier we proved that the strategies $(a_k, a_j)$ and $(a_j, a_k)$ are the PSNE of our evolutionary game. If two players play the same strategy i.e., play $\hat{p}, \hat{p}$ and are in equilibrium, then it is said to be a symmetric NE. Clearly, the PSNE of our game are not symmetric NE and by condition (1) of definition 3, $u(\hat{p}, \hat{p}) < u(p', \hat{p})$. Therefore, pure strategy NE is not evolutionarily stable according to definition 3. Another aspect of the PSNE is that it is always unfair for the player that selected the lower quality channel therefore making it impractical as a long term strategy for CRNs' channel selection.

*2) **Evolutionary Stability of MSNE:*** With no pure strategy NE for our evolutionary game as ESS, we now determine if the MSNE that we derived in equations (7) and (8) is an ESS according to definition 3. To do so, we first calculate $u(\hat{p}, \hat{p})$ i.e., see how the incumbent strategy $\hat{p}$ fares against itself and then determine the payoff of a mutant strategy $p'$ against the incumbent strategy. Consider the payoff matrix of table III where the players select strategies $a_k$ and $a_j$ with the probability distribution of the incumbent strategy $\hat{p} = \{\alpha, \beta\}$ then:

$$u(\hat{p}, \hat{p}) = \alpha\beta(u_k + u_j) \tag{10}$$

In equation (10) above, we have determined the payoff of incumbent strategy $\hat{p}$ when it competes against itself i.e., $u(\hat{p}, \hat{p})$. Now consider a mutant strategy $p' = \{\alpha + \delta, \beta - \delta\}$ which is greedier than the incumbent strategy $\hat{p}$ and assume that it selects the higher quality channel $k$ with a higher probability i.e., $\alpha + \delta$ and selects the lower quality channel $j$ with lower probability i.e., $\beta - \delta$, where $\delta$ is a small positive number that represents the increase in greediness/probability of a mutant strategy to select a higher quality channel. Because of the existence of two competing strategies, we now calculate $u(p', \hat{p})$ i.e., the utility of the mutant strategy against the incumbent strategy:

$$\implies u(p', \hat{p}) = \alpha\beta(u_k + u_j) - \delta(\alpha u_k - \beta u_j) \tag{11}$$

Since $u_k > u_j$ as assumed in section IV-B, we know that $\alpha u_k$ is greater than $\beta u_j$ and therefore the second term of equation (11) is positive. From equations (10) and (11) we have $u(\hat{p}, \hat{p}) > u(p', \hat{p})$. Since $u(\hat{p}, \hat{p})$ is strictly greater than $u(p', \hat{p})$, we do not need to check for the second condition of definition 3 and we conclude that the incumbent strategy $\hat{p}$ does strictly better than the mutation $p'$, which will die out in the evolutionary game. Hence our MSNE cannot be invaded by the greedier mutation $p'$ and is therefore an ESS.

It is pointed out that derivation of MSNE becomes intractable when the number of channels is greater than 2.

To expand our analysis for a $K$-channel scenario, we now introduce the concept of replicator dynamics in the following subsection.

### *E. Replicator Dynamics and K-Channel Scenario*

In the above section, we have shown that the mixed strategy NE of our proposed evolutionary game framework is evolutionarily stable. Evolutionary stability has provided us with a means to evaluate how the channel selection strategies perform in the long run when the CRNs do not cooperate with each other. This concept is somewhat static in nature because it does not demonstrate the dynamics with which the strategies evolve and converge to an equilibrium state. Replicator Dynamics explain how players evolve their behaviors by learning through strategic interactions at every stage/generation of the game to reach the equilibrium state which is also evolutionarily stable. In order to show the dynamics and to extend our analysis to the $K$-channel scenario, we now derive the Replicator Dynamics of our evolutionary heterogeneous spectrum sharing game.

From section IV-C, let $\hat{p} = \{p_1, p_2, ..., p_k\}$ and $\sum_{j=1}^{|\mathcal{K}|} p_j = 1$ where $\hat{p}$ represents the strategy of selecting channel $k$ with probability $p_k$. Alternatively, we can also think of $p_k$ as the proportion of population that select channel $k$ at any given time. Furthermore, let $u_0$ be the initial fitness of every CRN and the average payoff of CRNs selecting channel $k$ at a given stage of the game be represented by the set $\mathcal{U} = \{u_1, u_2, ..., u_K\}$. Then payoff for a CRN selecting channel $k$ can be calculated as:

$$u_k = u_0 + \sum_{j=1}^{|\mathcal{K}|} p_k u(a_k, a_j), \forall k, j \in \mathcal{K} \tag{12}$$

where $u(a_k, a_j)$ is the fitness of a CRN that selects channel $k$ in a pairwise competition against a CRN that selects channel $j$. Let $\bar{u}$ be the total average payoff of the entire CRN population at any given time. Then $\bar{u}$ for the entire population of CRNs is given by:

$$\bar{u} = \sum_{n=1}^{k} p_n u_n, \forall n \in K \tag{13}$$

and probability $p'_k$ of a CRN selecting channel $k$ for the next stage/time slot of the game is given by:

$$p'_k = p_k + \frac{p_k(u_k - \bar{u})}{\bar{u}} \tag{14}$$

Equations (12)-(14) are the replicator dynamics of our evolutionary spectrum sharing game. The idea behind the replicator dynamics is that if selecting channel $k$ in the current time slot results in a higher average fitness for the CRNs that selected it than the overall fitness of the entire CRN population, then the proportion of CRNs selecting channel $k$ in the next time slot will increase. In definition 2 of section IV-C, we stated that probability distribution $\hat{p}$ which is the game's MSNE, also represents the proportions of the population adopting different strategies at any

**Algorithm 1:** Replicator Dynamics Algorithm

**Data**: $u_0$, set of available channels $\mathcal{K}$ and their utilities $\mathcal{U}$
**Result**: Channel selection strategies converge to ESS.
**Initialization:** initial fitness of CRNs $u_0$, population distribution $p_k$, channel utilities $u_k$;

**for** *every stage/time step of the game* **do**
  With current channel utilities, compute average payoff $u_k$ for the proportion of CRN population that selected channel $k$ at current time - eq (12);
  Compute total average payoff $\bar{u}$ for the entire CRN population at current time - eq (13);
  Calculate new Channel selection strategies of CRNs - eq (14);
**end**

given stage of the game. CRNs are able to calculate the total average payoff for the entire CRN population $\bar{u}$ of eq (13) because it is based on common knowledge parameters: $p_n$ is the proportion of population that selected channel $n$ while channel quality represented by $u_n$ is also known to every CRN. In general, if selecting a particular channel in a given time slot results in a higher than total average payoff then that channel will be selected more frequently in subsequent time slots, ultimately converging to ESS.

## V. Fairness Analysis of Derived Equilibria

We now provide an analysis on the fairness of the Nash equilibria derived in preceding section. For the sake of clarity and ease of understanding, we consider the case of a 2-channel heterogeneous spectrum sharing game while the same arguments can be applied for analyzing a $K$-channel scenario. The Nash equilibria being considered as solutions for the spectrum sharing heterogeneous game are:

- Two pure-strategy NE for the anti-coordination game are $(a_k, a_j)$ and $(a_j, a_k)$.
- A mixed strategy NE defined by the probability distribution $\hat{p} = \{\alpha, \beta\}$ given by equations (7) and (8).

One of the ways to determine if entities receive a fair share of the system's resources is with Jains fairness index [23]. If there are $N$ CRNs and every CRN's utility is given as $u_i$ then fairness of the derived Nash equlibria can measured by Jain's equation as:

$$\mathcal{J}(u_1, u_2, ..., u_N) = \frac{\left(\sum_{i=1}^{N} u_i\right)^2}{N.\sum_{i=1}^{N} u_i^2} \tag{15}$$

As assumed previously in section III for a 2-channel scenario, channel $k$ is of higher quality than channel $j$ therefore $u_k > u_j$. Then from the payoff matrix of table II, gaining access to channel $k$ brings a larger payoff to a CRN whereas being of comparatively lower quality, channel $j$ brings a smaller payoff. There are two pure-strategy Nash equilibria $(a_k, a_j)$ and $(a_j, a_k)$, however intuitively, both of them are unfair because $u_k \neq u_j$ and one player always gets a smaller payoff than the other. This can be confirmed with eq (15) as follows: whenever all $u_i$ are equal then the ratio $(\sum_{i=1}^{N} u_i)^2 / \sum_{i=1}^{N} u_i^2$ in eq (15) yields a value equal to $N$ and Jain's index would be equal to 1 i.e., the maximum, while for an unequal distribution of payoffs it would be smaller than 1. Since PSNE does not result in equal payoff for all CRNs, it is not a fair solution.

Let us now consider fairness of MSNE. According to definition 2, MSNE is a probability distribution over the set of strategies which makes the players indifferent about their choice of strategies by making the payoffs equal even though the channels are of different quality. When all the payoffs $u_i$ become equal then from the same argument of the preceding paragraph, eq (15) yields an index equal to 1 resulting in the MSNE's resource distribution to be fair.

## VI. Simulations and Results

### A. Simulation Preliminaries

We have conducted simulations to study the effects of applying evolutionary game theoretic model for self-coexistence with heterogeneous channels and to study how the channel selection strategies in mixed strategy Nash Equilibria are also the evolutionarily stable states. We first show the results of simulations in which the collocated CRNs have only two available channels for which they contend and converge to an evolutionary stable state. Later, we show that our evolutionary game converges to ESS when there are more than 2 channels available for contention. To that end, we have implemented the Replicator dynamics and provide results of our experiments with 3, 4 and 5 heterogeneous channels as well. We also show that the evolutionary game can converge to new ESS when the network conditions may be changing requiring that the CRNs adjust to the new environments. As described in section IV-B, $a_k$ means the action of selecting channel $k$.

### B. Results

Figure 2 represents the scenario in which CRNs contend for 2 channels for secondary access. Figure 2a shows how CRNs select one out of two available channels with some probability where channel 1 is of better quality than channel 2. Any positive values for channel utilities would work however in case of simulations of figure 2 are assumed to be $u_1 = 9$ and $u_2 = 7$ for channels 1 and 2 respectively and its MSNE is $p_1 = 0.5625, p_2 = 0.4375$. Payoff from such strategic interactions is shown in figure 2b based on which, CRNs modify the probabilities of selecting the same channels in subsequent time slots/stages.

Let us first consider payoffs of CRNs that select channels with smaller payoffs. As shown in figure 2b, CRNs that select the lower quality channel receive a larger average payoff at $t = 1$ than CRNs that select higher quality channel. This happens because more CRNs would want to gain access to higher quality channel resulting in collisions and a zero payoff. Receiving higher payoff makes the CRNs that selected smaller payoff channels to further increase

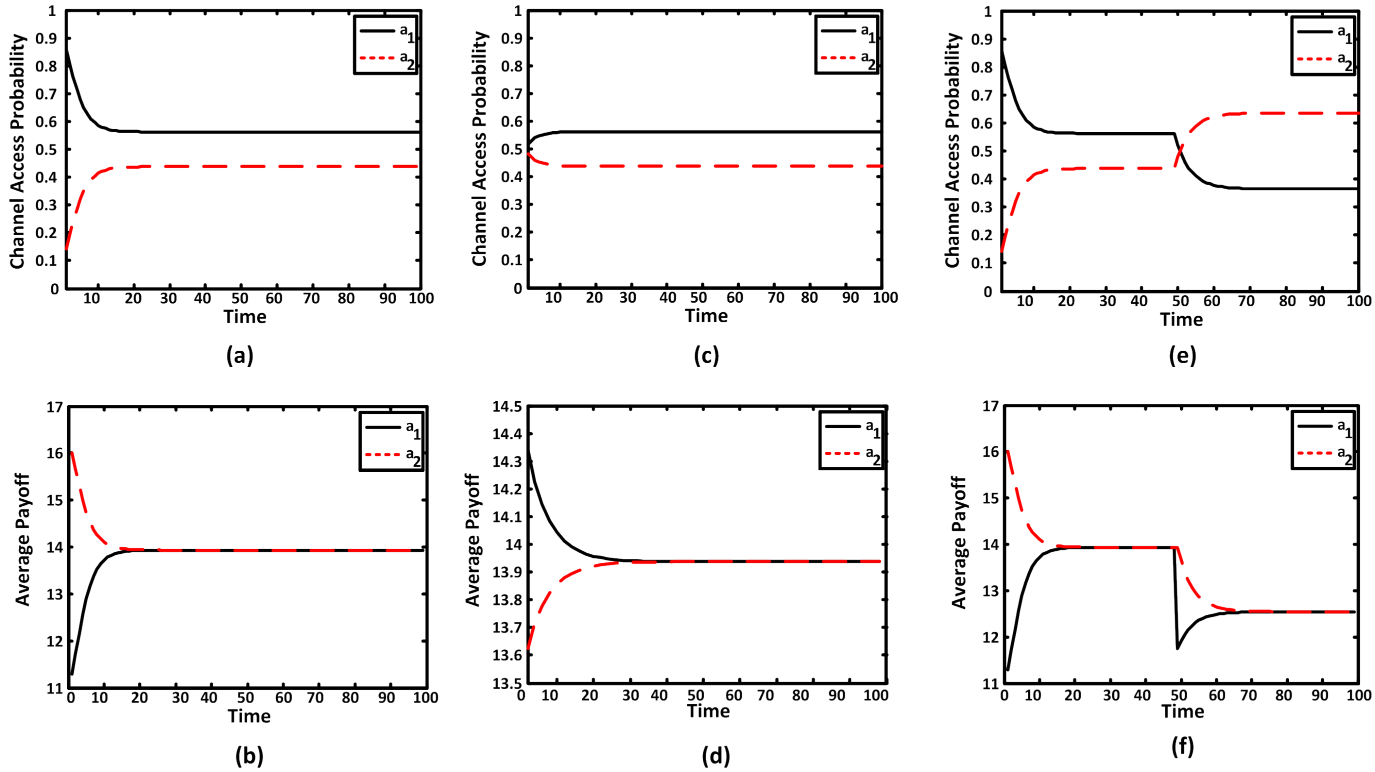


Fig. 2: Channel access probabilities and average payoffs when the number of channels available for contention is $K = 2$. (a) Channel access probability and (b) average payoffs when the initial probabilities are unequal (c) and (d) initial probabilities are equal, (e) and (f) under changing network conditions i.e., quality of channel 1 becomes worse than channel 2 at $t = 50$.

the probability of selecting the lower quality channel at $t = 2$ (2a). This however, results in lower average payoff for them at $t = 2$ than at $t = 1$. This happens because the higher quality channels are accessed with a relatively smaller probability at $t = 2$ because in previous time slot, it had resulted in smaller payoff. A relatively smaller payoff at $t = 2$ compared with higher payoff at $t = 1$ from accessing channel 2 is still greater than the total average payoff of the entire CRN which results in an even greater probability of selecting lower quality channel in subsequent stages. A similar yet opposite pattern can be seen for CRNs that select higher quality channels with higher probabilities. Stated in another way, the proportion of CRNs selecting a particular channel increases if its payoff is bigger than total average payoff of the entire population and vice versa.

CRNs keep modifying their channel selection probabilities in the same manner until their payoffs converge and they reach the ESS, which in the case of figure 2a is $p_1 = 0.5625, p_2 = 0.4375$ at around $t = 25$. The amount of time taken to converge to ESS is important as it would determine spectrum wastage because of collisions and is demonstrated in subsequent simulations. The average payoff $u_k$ of selecting a given channel $k$ is calculated by having the initial payoff $u_0$ of eq (12) equal to 1. Figures 2c and 2d represent the case when initial channel selection probabilities are equal yet they still converge to ESS. Figures 2e and 2f represent changing network conditions i.e., quality of channel 1 becomes worse than channel 2 at $t = 50$ yet the channel selection strategies still converge to

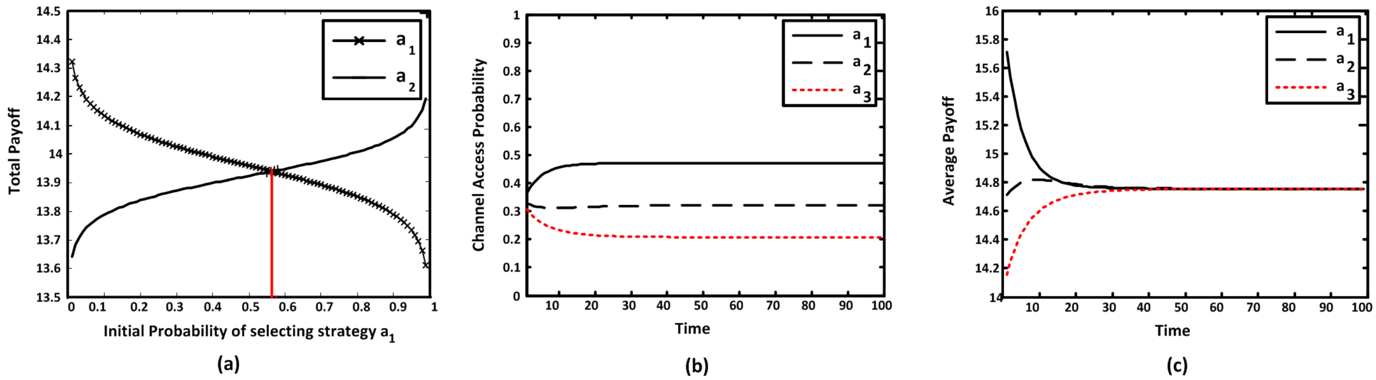


Fig. 3: (a) Total payoff from both channels becomes equal when initial probability of selecting channel 1 equals MSNE $p_1 = 0.5625$ i.e., the ESS probability. (b) Channel access probability and (c) average payoffs when the initial probabilities are equal for a 3-channel scenario.

a new ESS.

Figure 3a demonstrates that the MSNE of the evolutionary game achieves a *fair* distribution of the heterogeneous spectrum resources. For this simulation, there are two channels available for contention i.e., $K = 2$ and $u_1 = 9$ and $u_2 = 7$. As shown in figures 2a and 2c, CRNs are free to select any initial probabilities for the two channels but any selection results in convergence to ESS however the payoffs vary with every probability distribution. For the purpose of figure 3, the X-axis represents the initial probability of players selecting channel 1 i.e., at the start of simulation whereas every data point on the Y-axis represents total payoffs from selecting different probability distributions for channel selection until they reach ESS. With the given utilities, MSNE of the game is $p_1 = 0.5625$ and $p_2 = 0.4375$. The figure shows that the total payoff for both channels becomes equal when probability of selecting channel 1 equals $p_1 = 0.5625$ and therefore $p_2 = 0.4375$ which is the game's MSNE as well as the ESS as shown in figure 2 making it the only probability distribution of selecting the two channels that is *fair*.

We have also carried out simulations to demonstrate the robustness of our game to evolve an ESS even when the initial estimates of the players regarding heterogeneous channels are incorrect. To do so, we initialize the players's probabilities of accessing the channels to *equal* as well as *unequal* values and show that the strategies still converge to ESS. Also, we show that the game evolves its ESS even when the number of available channels is varied arbitrarily. However, the rate of convergence to ESS depends on the number of channels, difference in their relative quality and the accuracy of players estimates about their quality depicted by their choice of assigning initial access probabilities.

Figures 3b, 3c, 4, 5 and 6 show the convergence of channel selection probabilities to ESS along with their respective average payoffs in cases where the number of channels is increased to 3, 4 and 5 respectively and channel utilities are varied between values such as 9 and 4. The initial channel selection probabilities may be equal or unequal, yet in any case the game always converges to the ESS for any given set of channel utilities. Another

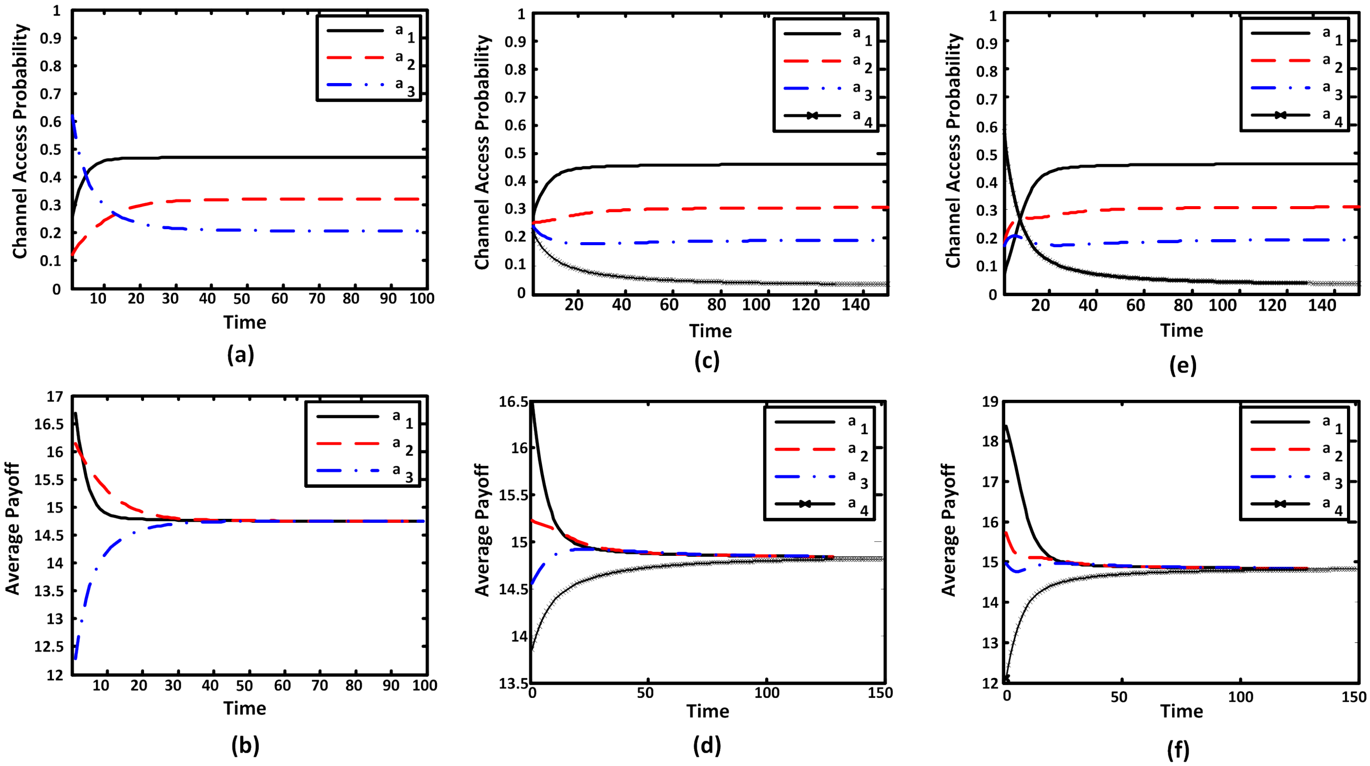


Fig. 4: Channel access probabilities and average payoffs when the number of channels available for contention is $K = 3$ and $K = 4$. (a),(e) Channel access probability and (b),(f) average payoffs when the initial probabilities are *unequal*, (c) Channel access probability and (d) average payoffs when the initial probabilities are *equal*.

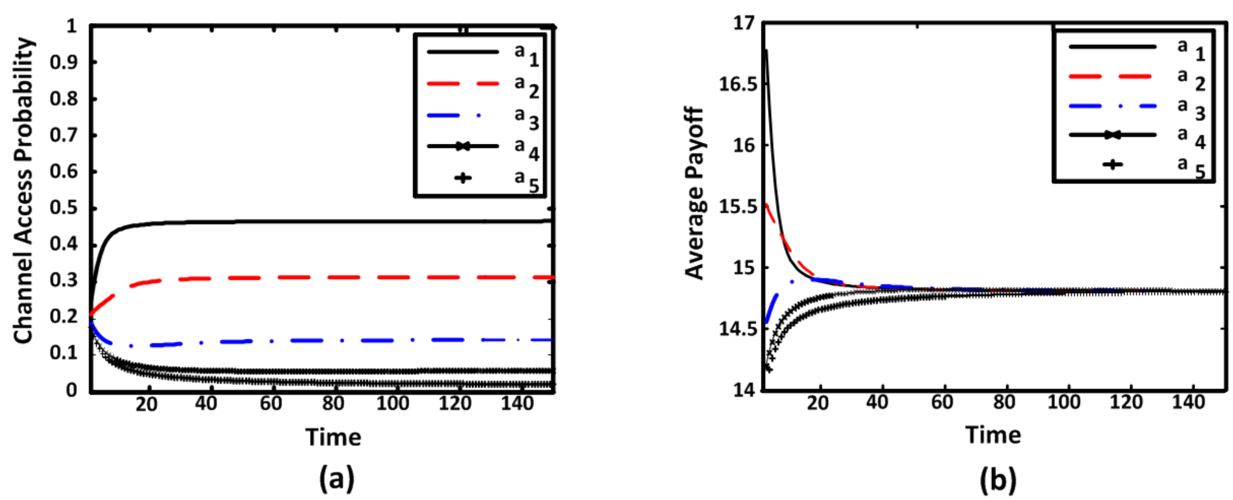


Fig. 5: Channel access probabilities and average payoffs when the number of channels available for contention is $K = 5$. (a) Channel access probability and (b) average payoffs when the initial probabilities are *equal*.

important observation is that the convergence rate to ESS decreases with the increase in number of channels and how accurate the initial probabilities are as compared to the ESS.

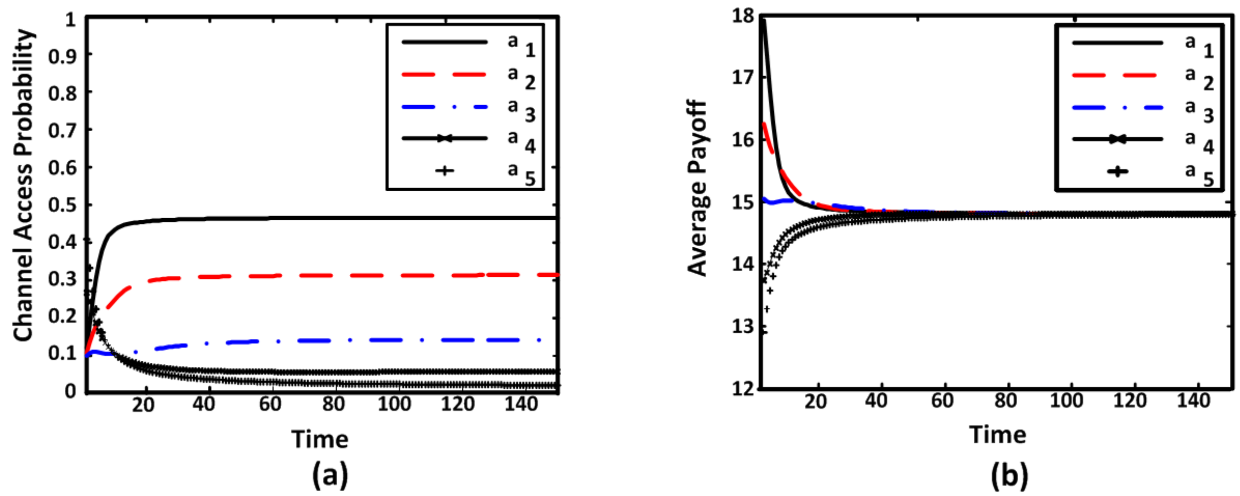


Fig. 6: Channel access probabilities and average payoffs when the number of channels available for contention is $K = 5$. (a) Channel access probability and (b) average payoffs when the initial probabilities are *unequal*.

## VII. Conclusion

Coexistence protocols employed by CRNs do not take into consideration the fact that spectrum bands vary significantly with regards to channel *quality* thereby making some channels of the spectrum bands more attractive to CRNs than others. In this paper, we aimed at answering the fundamental question of how CRNs should share heterogeneous spectrum bands in a distributed yet *fair* manner and proposed an evolutionary game theoretic framework to achieve that. We derived equilibrium strategies for CRNs spectrum sharing game for selecting particular spectrum bands and proved that the mixed strategy Nash Equilibria derived in the process are evolutionarily stable strategies (ESS) while also being fair. We also derived the mechanism of Replicator Dynamics with which players learn from payoff outcomes of their strategic interactions and modify their strategies at every stage of the evolutionary game. Since all players approach the ESS based solely upon the common knowledge payoff observations, our proposed evolutionary framework can be implemented in a distributed manner.

## References

[1] M. Faisal Amjad, M. Chatterjee, O. Nakhila, C.C. Zou, *"An Evolutionary Game Theoretic Framework for Coexistence in Cognitive Radio Networks"*, IEEE Global Conference on Signal and Information Processing (GlobalSIP) 2014.

[2] U.S. FCC, ET Docket 04-186, *"Notice of proposed rule making, in the matter of unlicensed operation in the TV broadcast bands,"* May 25, 2004.

[3] T. M. Taher, R. B. Bacchus, K. J. Zdunek, D. A. Roberson, *"Long-term spectral occupancy findings in Chicago,"* IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN), 2011.

[4] IEEE 802.22TM 2011 Standard for Wireless Regional Area Networks in TV Whitespaces, http://www.ieee.org/22

[5] X. Feng, H. Wang, and X. Wang, *"A game approach for cooperative spectrum sharing incognitive radio networks"* Wireless Communications and Mobile Computing, Volume 15, Issue 3, February 2015.

[6] H. Al-Tous, I. Barhumi, *"Resource Allocation for Multiple-Users AF-OFDMA Systems Using the Auction Framework,"* IEEE Transactions on Wireless Communications, Volume 14, Issue 5, 2014.

[7] B. Wang, K. J. R. Liu, T. C. Clancy, *"Evolutionary cooperative spectrum sensing game: how to collaborate?"* IEEE Transactions on Communications, Volume 58, Issue 3, March 2010.

[8] P. Sroka, A. Kliks, *"Distributed Interference Mitigation in Two-Tier Wireless Networks Using Correlated Equilibrium and Regret-Matching Learning,"* European Conference on Networks and Communications (EuCNC), 2014.

[9] J. Zheng, Y. Cai; D. Wu, *"Subcarrier allocation based on correlated equilibrium in multi-cell OFDMA systems"*, EURASIP Journal on Wireless Communications and Networking, Volume 2012:233, July 2012.

[10] J. W. Huang, V. Krishnamurthy, *"Cognitive Base Stations in LTE/3GPP Femtocells: A Correlated Equilibrium Game-Theoretic Approach,"* IEEE Transactions on Communications, Volume 59, Issue 12, October 2011.

[11] S. Maharjan, Y. Zhang, C. Yuen, S. Gjessing, *"Distributed Spectrum Sensing in Cognitive Radio Networks with Fairness Consideration: Efficiency of Correlated Equilibrium,"* IEEE Mobile Adhoc and Sensor Systems (MASS), 2011.

[12] B. Wang, Z. Han, K.J.R. Liu, *"Peer-to-peer file sharing game using correlated equilibrium,"* 43rd Annual Conference on Information Sciences and Systems, IEEE CISS 2009.

[13] S. Sengupta, R. Chandramouli, S. Brahma, M. Chatterjee, *"A game theoretic framework for distributed self-coexistence among IEEE 802.22 networks,"* IEEE GLOBECOM 2008.

[14] R. Etkin, A. Parekh, D. Tse, *""Spectrum sharing for unlicensed bands,"* IEEE Journal on Selected Areas in Communications (JSAC), Volume 25, Issue 3, April 2007.

[15] C. Jiang, Y. Chen, Y. Gao, K. J. Ray Liu, *"Joint spectrum sensing and access evolutionary game in cognitive radio networks,"* IEEE Transactions on Wireless Communications, Volume 12, Issue 5, May 2013.

[16] M. Faisal Amjad, M. Chatterjee, C.C. Zou, *"Inducing Voluntary Cooperation for Optimal Coexistence in Cognitive Radio Networks: A Game Theoretic Approach"*, IEEE Military Communications Conference (Milcom) 2014.

[17] S. Sengupta, S. Brahma, M. Chatterjee, N. Shankar, *"Self-coexistence among interference-aware IEEE 802.22 networks with enhanced air-interface,"* Pervasive and Mobile Computing, Volume 9 Issue 4, August 2013.

[18] Z. Han, C. Pandana, K.J.R. Liu, *"Distributive Opportunistic Spectrum Access for Cognitive Radio using Correlated Equilibrium and No-Regret Learning,"* Wireless Communications and Networking Conference, IEEE WCNC 2007.

[19] Google, Inc.'s TV Bands Database System for Operation, ET Docket No. 04-186 http://www.google.com/get/spectrumdatabase/channel/

[20] Show My White Space TVWS database from Spectrum Bridge Inc. http://whitespaces.spectrumbridge.com/whitespaces/home.aspx

[21] D. Fudenburg, J. Tirole, *"Game Theory,"* The MIT press, 1991.

[22] J. Maynard Smith *"Evolution and the theory of games"*, Cambridge University press, 1982.

[23] R. Jain, W. Hawe, D. Chiu, *"A Quantitative measure of fairness and discrimination for resource allocation in Shared Computer Systems"*, DEC-TR-301, September 26, 1984.

[24] Y. Shoham, K. Leyton-Brown, *"Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations"*, Cambridge University Press 2008.