

UCF



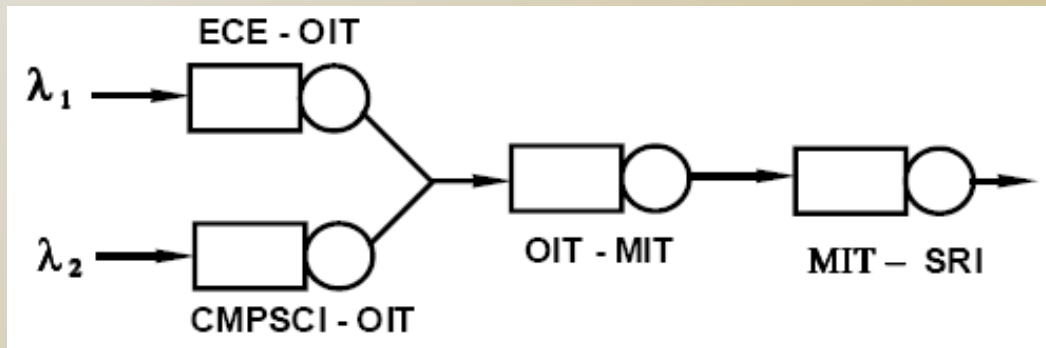
Stands For Opportunity

CDA6530: Performance Models of Computers and Networks

Chapter 7: Basic Queuing Networks

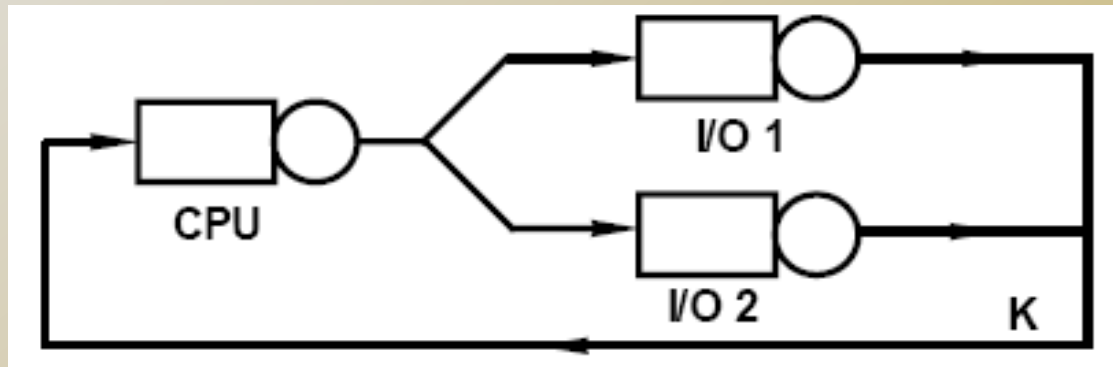
Open Queuing Network

- Jobs arrive from external sources, circulate, and eventually depart



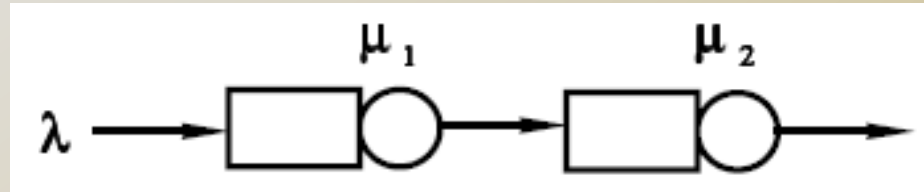
Closed Queuing Network

- Fixed population of K jobs circulate continuously and never leave
 - Previous machine-repairman problem



Feed-Forward QNs

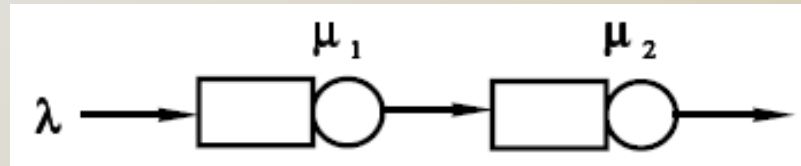
- Consider two queue tandem system



- Q: how to model?
 - System is a continuous-time Markov chain (CTMC)
 - State $(N_1(t), N_2(t))$, assume to be stable
 - $\pi(i,j) = P(N_1=i, N_2=j)$
 - Draw the state transition diagram
 - But what is the arrival process to the second queue?

Poisson in \Rightarrow Poisson out

- ❑ **Burke's Theorem:** Departure process of $M/M/1$ queue is Poisson with rate λ independent of arrival process.
- ❑ **Poisson process addition, thinning**
 - ❑ Two *independent* Poisson arrival processes adding together is still a Poisson ($\lambda = \lambda_1 + \lambda_2$) **Why?**
 - ❑ For a Poisson arrival process, if each customer leaves with prob. p , the remaining arrival process is still a Poisson ($\lambda = \lambda_1 \cdot p$)



- State transition diagram: (N_1, N_2) , $N_i=0,1,2,\dots$

$$\pi(i, j) = (1 - \rho_1)\rho_1^i(1 - \rho_2)\rho_2^j \quad i, j \geq 0$$

$$\rho_i = \lambda/\mu_i$$

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- ❑ For a k queue tandem system with Poisson arrival and expo. service time
 - ❑ Jackson's theorem:

$$P(N_1 = n_1, N_2 = n_2, \dots, N_k = n_k) = \prod_{i=1}^k (1 - \rho_i) \rho_i^{n_i},$$

- ❑ Above formula is true when there are feedbacks among different queues
 - ❑ Each queue behaves as M/M/1 queue in isolation

Example

- λ_i : arrival rate at queue i

$$\lambda_1 = 4 + \lambda_2/4$$

$$\lambda_2 = 5 + \lambda_1/2$$

Why?

$$\Rightarrow \lambda_1 = 6, \lambda_2 = 8$$

$$\pi(n_1, n_2) = \frac{1}{4} \left(\frac{3}{4}\right)^{n_1} \frac{1}{5} \left(\frac{4}{5}\right)^{n_2}$$

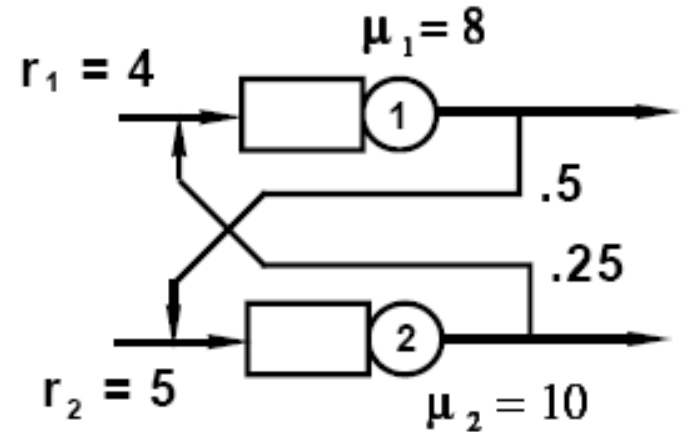
In M/M/1:

$$E[N] = \frac{\rho}{1 - \rho} = \frac{\lambda}{\mu - \lambda}$$

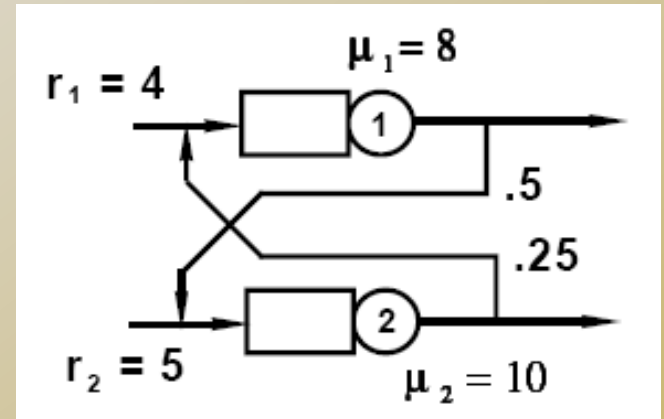
$$\begin{aligned} E[N] &= \sum_{i=1}^2 E[N_i] = \sum_{i=1}^2 \lambda_i / (\mu_i - \lambda_i) \\ &= 3 + 4 = 7 \end{aligned}$$

$$E[T] = E[N] / (r_1 + r_2) = 7/9 \text{ time units}$$

Why?



- $T^{(i)}$: response time for a job enters queue i



$$E[T^{(1)}] = 1/(\mu_1 - \lambda_1) + E[T^{(2)}]/2$$

$$E[T^{(2)}] = 1/(\mu_2 - \lambda_2) + E[T^{(1)}]/4$$

Why?

In M/M/1: $E[T] = \frac{1}{\mu - \lambda}$

Extension

- Results hold when nodes are multiple server nodes ($M/M/c$), infinite server nodes finite buffer nodes ($M/M/c/K$) (careful about interpretation of results), PS (process sharing) single server with arbitrary service time distr.

Closed QNs

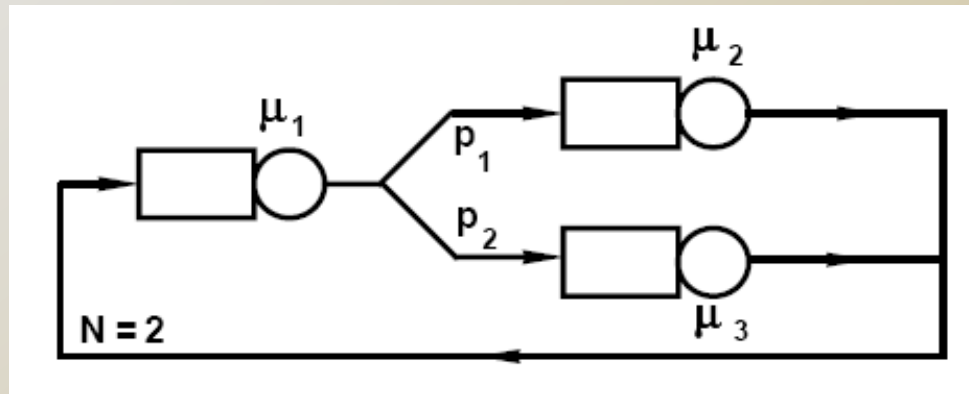
- Fixed population of N jobs circulating among M queues.
 - single server at each queue, exponential service times, mean $1/\mu_i$ for queue i
 - routing probabilities $p_{i,j}$, $1 \leq i, j \leq M$
 - visit ratios, $\{v_i\}$. If $v_1 = 1$, then v_i is mean number of visits to queue i between visits to queue 1

$$v_i = \sum_{j=1}^M v_j p_{j,i} \quad i = 2, \dots, M$$

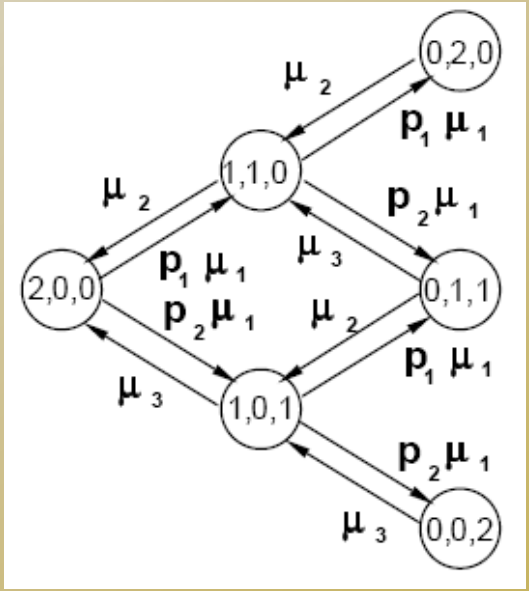
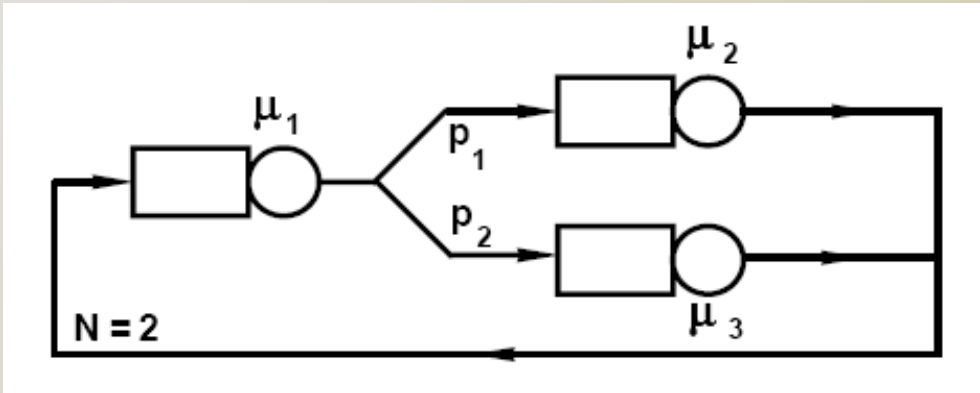
- γ_i : throughput of queue i ,

$$\gamma_i / \gamma_j = v_i / v_j, \quad 1 \leq i, j \leq M$$

Example



- ❑ Open QN has infinite no. of states
- ❑ Closed QN is simpler
- ❑ How to define states?
 - ❑ No. of jobs in each queue



Steady State Solution

□ Theorem (Gordon and Newell)

$$\pi(\vec{n}) = \frac{1}{G(N)} \prod_{i=1}^M \left(\frac{v_i}{\mu_i} \right)^{n_i} \quad \vec{n} \geq \vec{0}; \sum_{i=1}^M n_i = N$$

where $\vec{n} = (n_1, \dots, n_M)$, and $G(N)$ is a constant chosen so that $\sum \pi(\vec{n}) = 1$.

- For previous example when $p_1=0.75$, v_i ?

$$v_1 = 1, v_2 = 3/4, v_3 = 1/4$$

