Wednesday, September 02, 2015 12:02 PM

$$P(X=k)=e^{-\lambda}\frac{\lambda^k}{k!}$$

$$P(X=k)=\binom{n}{k}p^k(1-p)^{n-k} \qquad \binom{n}{k}=\frac{n!}{(n-k)!k!}$$

$\lambda = np$ $\rightarrow$ $p(X=k) = e^{-np}\dfrac{n^k \cdot p^k}{k!}$

$\binom{n}{k} \approx \dfrac{n^k}{k!}$

$(1-p)^{n-k} \approx e^{-np}$

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!} = \frac{n\cdot(n-1)\cdot(n-2)\cdots(n-k+1)}{k!}$$

if $k \ll n$,

$$\approx \frac{n^k}{k!}$$

$$e^x = \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n$$

if n is big enough

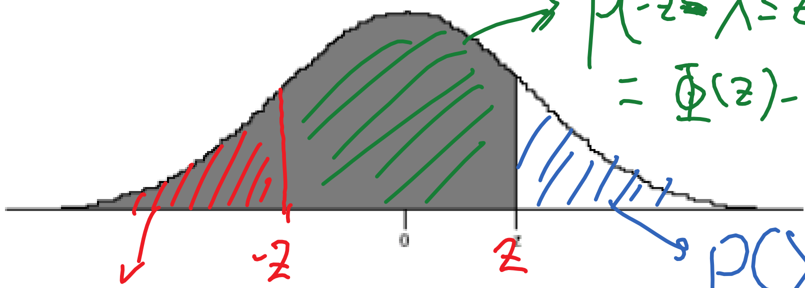
p is small enough

$$(1-p)^{n-k} \approx (1-p)^n \approx e^{-pn}$$

$F_X(x) \equiv P(X \le x) \equiv \Phi(x)$

$F_X(z) = P(X \le z)$

Wednesday, September 02, 2015 12:39 PM



$P(-z \le X \le z)$

$= \Phi(z) - \Phi(-z)$

if $z = 1$

$P(|X| \le z) = 0.68$

$P(X > z)$

$P(X \le -z) = P(X > z)$

$= 1 - P(X \le z)$

$\Rightarrow \Phi(-z) = 1 - \Phi(z)$

$\Rightarrow \Phi(-z) + \Phi(z) = 1$

Q1: $P(X \le 365)$? $X \sim N(300, 50^2)$ $\mu = 300$, $\sigma = 50$

Wednesday, September 02, 2015   12:56 PM

define r.v. $Z = \frac{X-\mu}{\sigma} = \frac{X-300}{50}$ then $Z \sim N(0,1)$

$\rightarrow X = 50Z + 300$

$P(X \le 365) = P(50Z+300 \le 365) = P\left(Z \le \frac{365-300}{50}\right) = P(Z \le 1.3)$
$= \Phi(1.3)$

by lookup std normal table $P(Z \le 1.3) = 0.903$

$\therefore P(X \le 365) = 0.903$

$P(\text{light bulb works after 1 year}) =$
$= 1 - P(X \le 365) = 0.097$

$np = 100 \times 0.097 = 9.7$