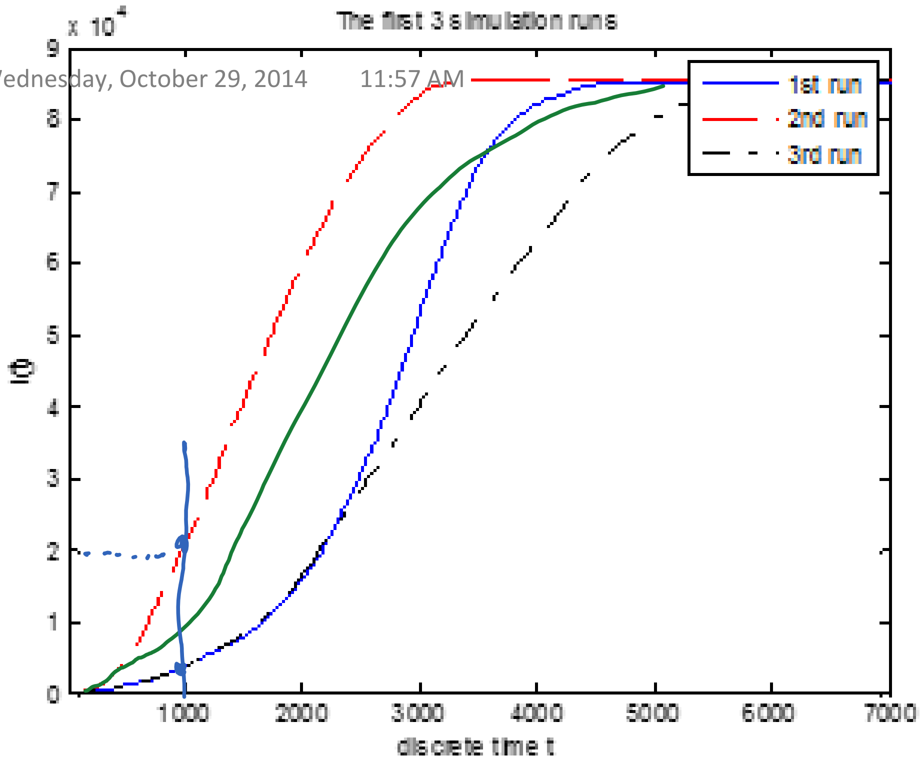
Wednesday, October 29, 2014 11:57 AM



mean $E[I(1000)]$?

red: $I(1000) = 20{,}000$

blue & black $I(1000) = 3000$

$$E[I(1000)] = \frac{20{,}000 + 3000 + 3000}{3}$$

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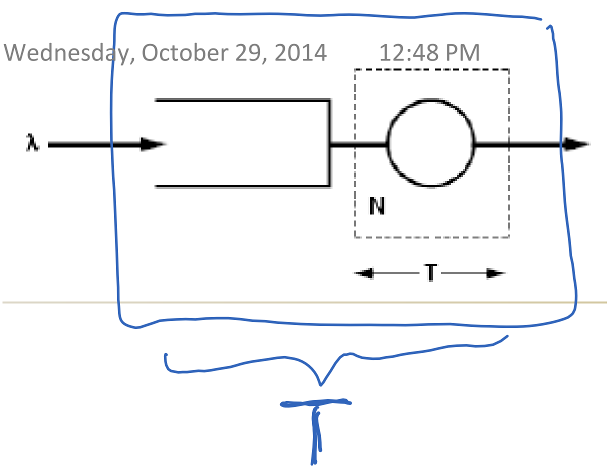

N

$\pi Q = 0$ and $\sum \pi_i = 1$

3-nodes

$$\pi = (\pi_0 \; \pi_1 \; \pi_2) \begin{bmatrix} & & \\ & Q & \\ & & \end{bmatrix} = [0 \;\; 0 \;\; 0]$$

$$(\pi_0 \; \pi_1 \; \pi_2) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = [1]$$

$$(\pi_0 \; \pi_1 \; \pi_1) \underbrace{\begin{bmatrix} 1 & Q(:,2:3) \\ 1 & \\ 1 & \end{bmatrix}}_{} = [1 \; \underbrace{0 \;\; 0}_{B}]$$

$$\pi \cdot Q1 = B$$

$$\pi = B \cdot Q1^{-1}$$

$$\Rightarrow \pi_n = \rho^n \pi_0$$

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$$\sum_{i=0}^{\infty} \pi_i = 1$$

$$\pi_0 + \pi_1 + \pi_2 + \cdots = 1$$

$$\Rightarrow \pi_0 + \rho\pi_0 + \rho^2\pi_0 + \cdots = 1$$

$$\Rightarrow \pi_0 (\underbrace{1 + \rho + \rho^2 + \rho^3 + \cdots}_{\text{if } \rho<1,\ \frac{1}{1-\rho}}) = 1$$

$$\Rightarrow \pi_0 = 1-\rho$$

$$\Rightarrow \quad \pi_0 = 1 - \rho$$

$$\rho \equiv \lambda/\mu$$

$$\pi_0 \sum_{k=1}^{\infty} k\rho^k = \frac{\rho}{1-\rho}$$

$$S = \rho + 2\rho^2 + 3\rho^3 + 4\rho^4 + \cdots$$

$$\to \rho S = \rho^2 + 2\rho^3 + 3\rho^4 + \cdots$$

$$(1-\rho)S = \rho + \rho^2 + \rho^3 + \rho^4 + \cdots = \frac{\rho}{1-\rho}$$

$$\Rightarrow S = \frac{\rho}{(1-\rho)^2}$$

$$E[N] = \pi_0 S = (1-\rho) \cdot \frac{\rho}{(1-\rho)^2}$$

$$= \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda}$$

$$E[N] \cdot E[X] = \frac{\lambda}{\mu-\lambda} \times \frac{1}{\mu}$$