CDA6530, lecture 17

Note Title 10/15/2013

$E[X] = \frac{1}{\mu}$ μ customers served per unit time

$\mu = 100$ $\lambda = 80$ $\rho = \frac{\lambda}{\mu} = 0.8$

Q1 $\rightarrow$ $E[N]$? $E[N] = \frac{\rho}{1-\rho} = \frac{0.8}{0.2} = 4$

Q2 $E[W]$? $E[W] = E[N] \cdot E[X] = 4 \times \frac{1}{100} = 0.04$ sec

Q3 $E[T]$? $E[T] = \frac{1}{\mu - \lambda} = \frac{1}{20} = 0.05$ sec

Q4 : $P(N=0) \equiv \pi_0$ $\pi_0 = 1 - \rho = 0.2$

Q5 : $P(N>5) \rightarrow \sum_{k=6}^{\infty} \pi_k = 1 - \sum_{k=0}^{5} \pi_k$

Q6 : find μ, such ~~that~~ that $E[T] = 0.02$

$\frac{1}{\mu - \lambda} = 0.02 \Rightarrow \mu = \lambda + 50 \Rightarrow \mu = 130$

bandwidth is 130 kbps

$e^{\alpha} = \sum_{n=0}^{\infty} \alpha^n / n!$

X_1, X_2, X_3

$P(X_i \le t) = 1 - e^{-\lambda t}$

$P(X_i > t) = e^{-\lambda t}$

$Y = \min(X_1, X_2, X_3)$

$$
\begin{aligned}
P(Y \le t) &= 1 - P(Y > t) \\
&= 1 - P(X_1 > t) \cdot P(X_2 > t) \cdot P(X_3 > t) \\
&= 1 - e^{-3\lambda t}
\end{aligned}
$$

$\Rightarrow Y \sim$ exp. distr. rate $= 3\lambda$

$$
\begin{cases}
\lambda \pi_{i-1} = i\mu\pi_i, & i \le c, \\
\lambda \pi_{i-1} = c\mu\pi_i, & i > c
\end{cases}
\longrightarrow
$$

$\lambda \pi_0 = \mu \pi_1 \Rightarrow \pi_1 = \rho \pi_0$

$\lambda \pi_1 = 2\mu \pi_2 \Rightarrow \pi_2 = \frac{1}{2} \cdot \rho \pi_1 = \frac{\rho^2}{2} \pi_0$

$\sum_{i=0}^{\infty} \pi_i = 1$

$\pi_i = \frac{\rho^i}{i!} \pi_0 \quad i \le c$

$$\lambda \pi_{i-1} = i\mu\pi_i,$$

$$\pi_i = \frac{\rho^i}{i!}\pi_0$$

$$\pi_i = \frac{\rho}{i}\pi_{i-1}$$

$$\sum_{i=0}^{\infty} \pi_i = 1$$

$$\Rightarrow \sum_{i=0}^{\infty} \frac{\rho^i}{i!}\pi_0 = 1$$

$$\Rightarrow \pi_0 \sum_{i=0}^{\infty} \frac{\rho^i}{i!} = 1$$

$$\pi_0 e^{\rho} = 1 \Rightarrow \pi_0 = e^{-\rho}$$