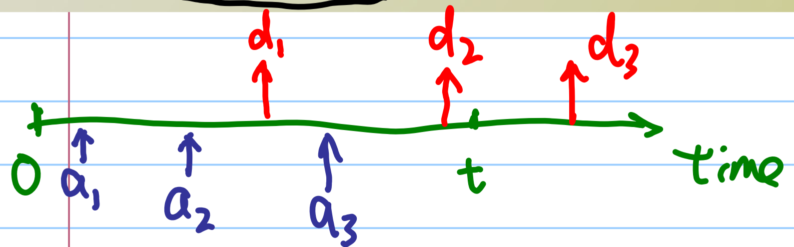
$$\gamma(t) = \sum_{n=1}^{\alpha(t)} \left(\min\{d_n, t\} - a_n\right) = \int_0^t N(s)ds$$



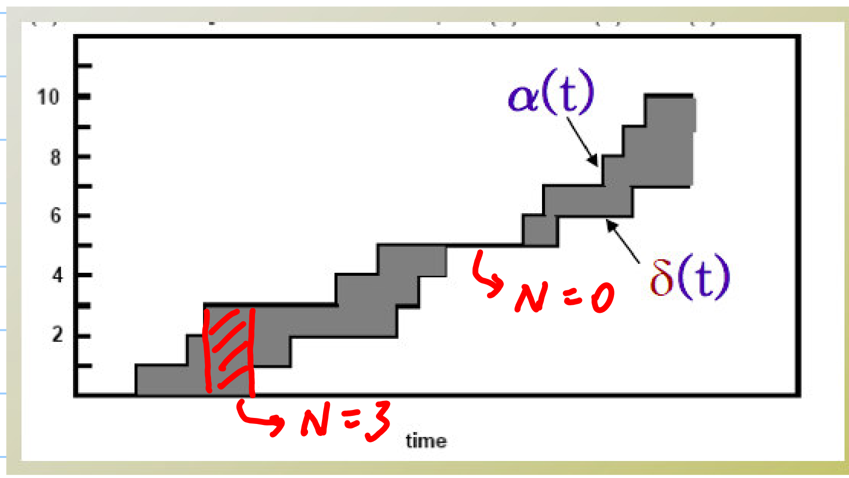
$\gamma(t) = (d_1 - a_1) + (d_2 - a_2) + (t - a_3)$

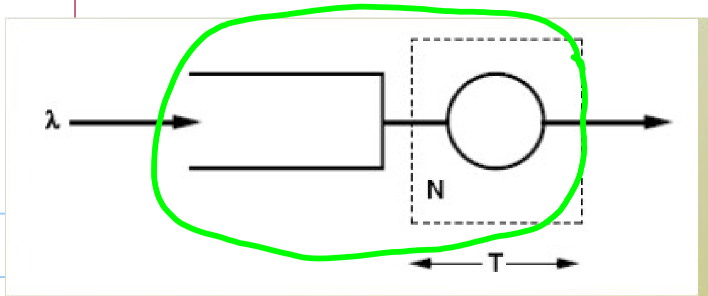
$N_t = \gamma(t)/t$

$T_t = \gamma(t)/\alpha(t)$

$\lambda_t = \alpha(t)/t$

$N_t = \dfrac{T_t \cdot \alpha(t)}{t} = \lambda_t \cdot T_t$





$N = \lambda T$

N: # of customers in system (queue or server)

$N \in [0, 1, 2, 3, \ldots\ldots]$

T: avg. time spent in the system

$$\pi_1 \lambda = \pi_2 \mu \quad \Rightarrow \pi_2 = \rho^2 \pi_0$$

$\pi_1 = \rho \pi_0$

$$\boxed{\sum_{i=0}^{\infty} \pi_i = 1}$$

$\hookrightarrow \pi_2 = \frac{\lambda}{\mu} \pi_1 = \rho \cdot \pi_1 = \rho^2 \pi_0$

$\pi_0 + \pi_1 + \pi_2 + \cdots\cdots = 1 \implies \pi_0 + \rho \pi_0 + \rho^2 \pi_0 + \cdots\cdots = 1$

$\implies \pi_0 \underbrace{(1 + \rho + \rho^2 + \rho^3 + \cdots\cdots)}_{\hookrightarrow \frac{1}{1-\rho}} = 1 \implies \pi_0 = 1 - \rho \implies \rho = 1 - \pi_0$

$\pi_n \equiv P(N = n)$

Prob. server is busy

$$\sum_{k=1}^{\infty} k\pi_k = \pi_0 \sum_{k=1}^{\infty} k\rho^k = \frac{\rho}{1-\rho}$$

$\pi_k = \pi_0 \rho^k \qquad \pi_0 = 1-\rho$

$E[N] = \pi_0 [\underbrace{\rho + 2\rho^2 + 3\rho^3 + \cdots}_{S = \frac{\rho}{(1-\rho)^2}})$

$S = \rho + 2\rho^2 + 3\rho^3 + 4\rho^4 + \cdots$

$\rightarrow \rho S = \rho^2 + 2\rho^3 + 3\rho^4 + \cdots$

$(1-\rho)S = \rho + \rho^2 + \rho^3 + \rho^4 + \cdots$

$= \frac{\rho}{1-\rho}$

$E[N] = (1-\rho) \cdot \frac{\rho}{(1-\rho)^2} = \frac{\rho}{1-\rho}$

D/D/1 :

$\rho = \frac{\lambda}{\mu}$

$E[W] = E[N] \cdot E[X] = \frac{\rho}{1-\rho} \cdot \frac{1}{\mu} = \frac{\lambda/\mu}{1-\lambda/\mu} \cdot \frac{1}{\mu} = \frac{\lambda}{\mu(\mu-\lambda)}$

$E[T] = \frac{1}{\mu} + \frac{\lambda}{\mu(\mu-\lambda)} = \frac{\mu-\lambda+\lambda}{\mu(\mu-\lambda)} = \frac{1}{\mu-\lambda}$