Project 5: Dog Catching Mailman Problem
Discrete Time Simulation
CDA 6530

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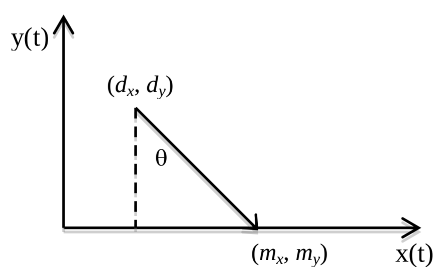
1. Model Description:

The model is based on discrete time simulation. It only simulates the action of dog and mailman at sharp time ticks. At the beginning of any tick, the dog will measure the position of its own location and the mailman's location and calculate an angel and a speed toward the mailman's position. Later in this time tick period, the dog will remain going along a straight line towards the point it aimed at the very beginning of the time tick. Meanwhile, the mailman will keep running along the x-axis and try to get off. When the dog reaches the x-axis, it will stop for the rest time during that particular time tick duration and in the following time, it will just chasing in the x-axis just the same as the mailman. For the type two dog, a randomly slip procedure will happen from time to time to refresh the dogs speed to zero, Nevertheless, the dog's accelerate will be larger than the type one dog.

2. The high-level algorithm:

The most significant algorithm in the project is how the dog selects the direction in the beginning of every time tick period. Supposing the current position of dog and mailman are (d_x, d_y) and (m_x, m_y) respectively,



We could calculate the angle of the dog's speed by the current position of dog and mailman as (1) ,(2) and (3)

$$D(d,t) = \sqrt{(m_x(t) - d_x(t))^2 + (m_y(t) - d_y(t))^2} \quad (1)$$

$$sin\,\theta = \frac{m_x(t) - m_x(0)}{D(d,m)} \quad (2)$$

$$cos\,\theta = \frac{d_y(t)}{D(d,m)} \quad (3)$$

Where $D(d,t)$ is the Euclidian distance of the dog and mailman at the beginning of each time tick. After calculating the speed angle θ, we could decompose the speed into x-axis and y-axis thus obtain the dog's position at the end of the time tick using (4) , (5) and (6).

$$d_x(t + \Delta t) = d_x(t) + \Delta t \cdot v_d(t) \cdot \sin\,\theta \quad (4)$$

$$d_y(t + \Delta t) = d_y(t) - \Delta t \cdot v_d(t) \cdot \cos\theta \quad (5)$$

$$m_x(t + \Delta t) = m_x(t) + \Delta t \cdot v_m(t) \quad (6)$$

Where Δt is the time step, (0.2 second in the experiment), $v_d(t)$ and $v_m(t)$ are the dog speed and mailman speed at time t respectively. Only one thing must be noticed that, when the dog reaches the x-axis, $d_y(t + \Delta t) < 0$, it will no longer remain going on the direction and cross the x-axis, rather, it will stop and wait until the next tick. It is described in (7)

$$\begin{cases} d_y(t + \Delta t) = 0 \\ d_x(t + \Delta t) = m_x(t) \end{cases} \tag{7}$$

3. Start and termination condition:

The start condition is

$$\begin{cases} d_x(0) = 0 \\ d_y(0) = 20 \\ m_x(0) = 10 \\ m_y(0) = 0 \\ v_d(0) = 0 \\ v_m(0) = 0 \end{cases} \tag{8}$$

The termination condition is

$$\begin{cases} d_y(t) = 0 \\ d_x(t) \geq m_x(t) \end{cases} \tag{9}$$

4. Experimental results:

Type 1 dog and type 1 mailman:

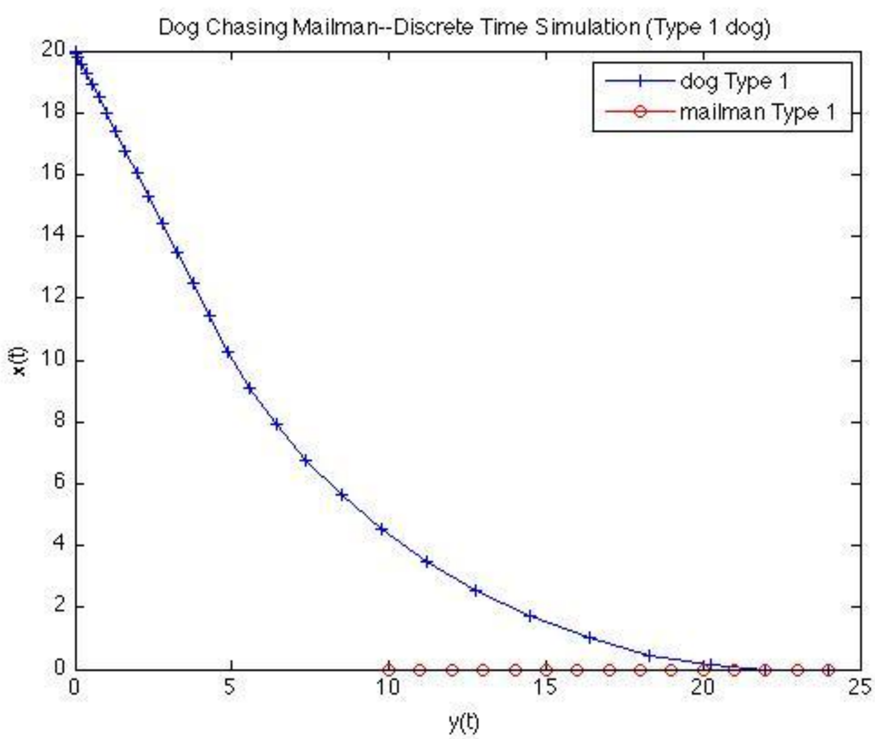


Fig.1: Type 1 dog chasing Type 1 mailman

Type 1 dog and type 2 mailman:

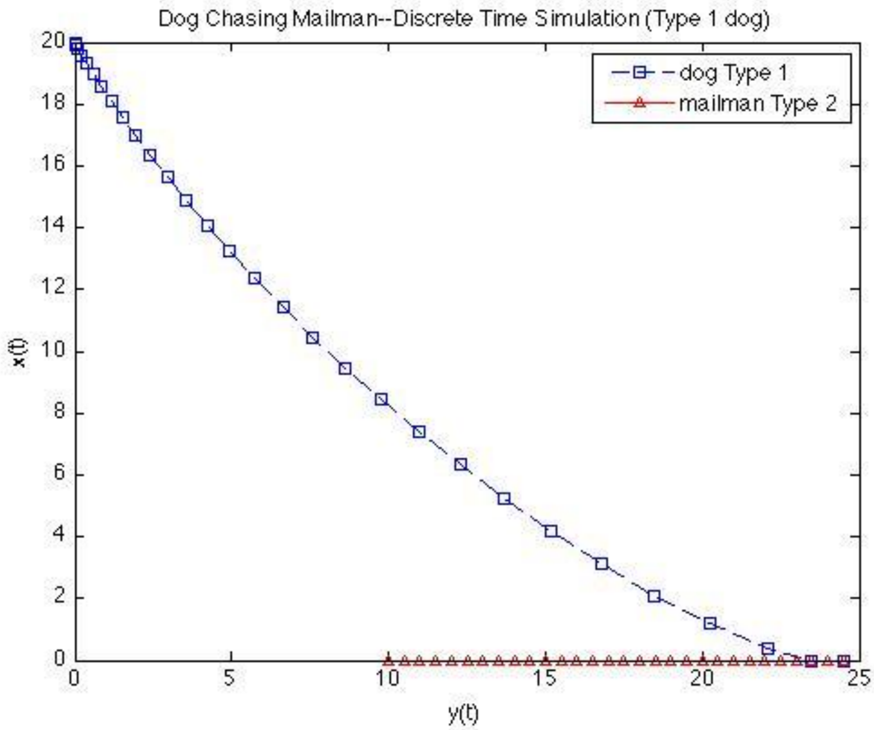


Fig.2: Type 1 dog chasing Type 2 mailman

Type 2 dog and type 1 mailman (first 3 runs):

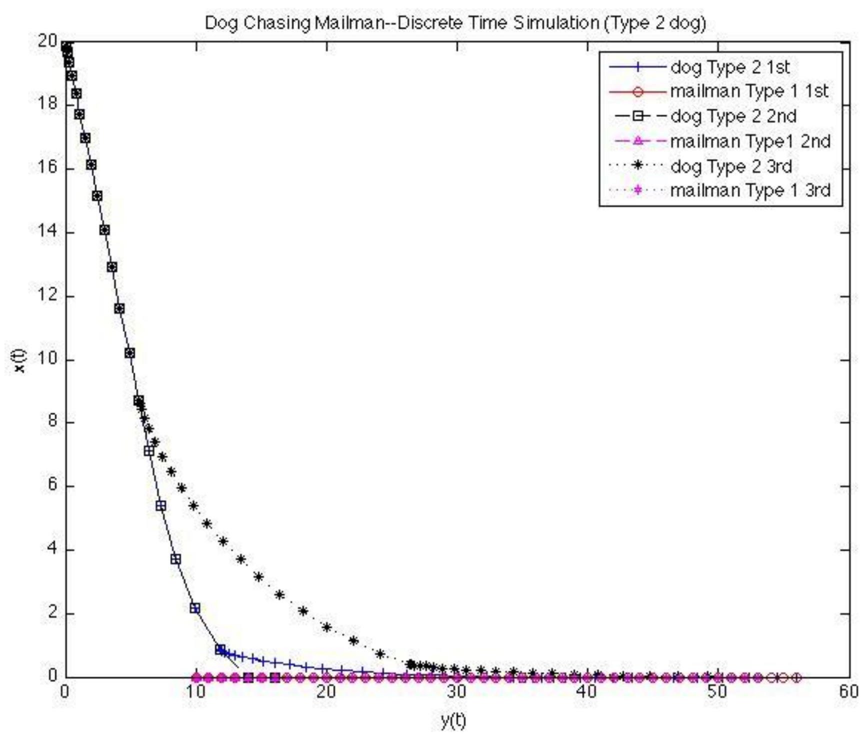


Fig.3: Type 2 dog chasing Type1 mailman

Type 2 dog and type 2 mailman (first 3 runs):

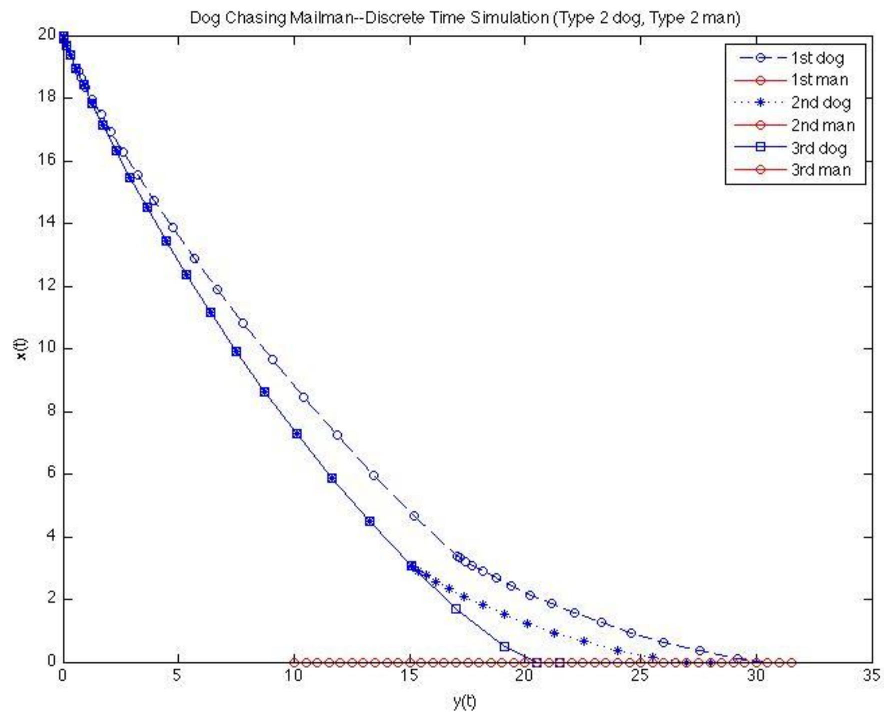


Fig.4: Type 2 dog chasing Type2 mailman

All 4 cases together:

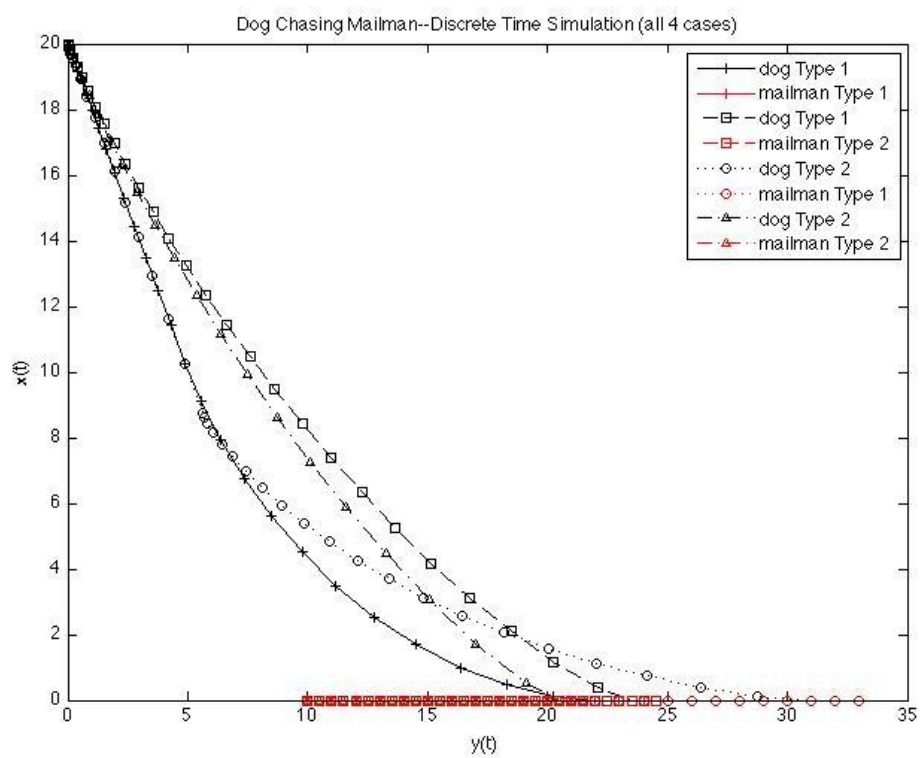


Fig.5: Discrete time simulation results of all 4 cases

5. Comparison and analysis of the experimental results

The final catching time, final catching position, dog's position at half time and mailman's position at half time are summarized in the following table,

Tab.1: comparison of the catching results between 4 cases

	Final catch time (sec)	Final catch pos (x(t),y(t))	Dog pos half T (x(t),y(t))	Man pos half T (x(t),y(t))
1-dog, 1-man	5.8	(24,0)	(3.7566,12.4868)	(10,0)
1-dog, 2-man	5.8	(24.5,0)	(4.951,13.2369)	(17.0)
2-dog, 1-man	6.112	(25.56,0)	(6.0931,10.9304)	(13.33,0)
2-dog, 2-man	5.636	(24.09,0)	(6.276, 11.6001)	(16.795,0)

6. Conclusion

It can be noted in Tab.1 that, for type 1 dog, catching time is the same, no matter whether the mailman will freeze for 3 seconds to tie shoe or not. The final catching position for type two man will be slightly longer than type 1 man, therefore, to survive longer, it is wise not to tie the shoe, but run directly.

For scenarios in type 2 dog, the results are generated based on the average of 100 runs. It is obvious that results in type 2 dog are more easily to be distinguishable between type 1 man and type 2 man. Type 1 man can run longer distance and survive longer time before being caught by the type 2 dog. The result is different from the type 1 dog probably because that the type 2 dog will slip and slow to zero by some probability, although their acceleration is faster, the actual average speed during the chasing phase can be relatively smaller due to the frequent slip. In that case, the mailman's speed can attach greater significance in the entire process. Therefore, for the type 2 dog, it is wise for the mailman to spend time to tie the shoe and ran faster.