CDA6530: Performance Models of Computers and Networks

Chapter 9:Statistical Analysis of Simulated Data and Confidence Interval

Sample Mean

- R.v. X: E[X]=θ, Var[X]=σ^2
- Q: how to use simulation to derive?
  - Simulate X repeatedly
- $X_1, \cdots, X_n$ are i.i.d., $=_{\text{statistic}}$ X
- Sample mean: $\bar{X} \equiv \sum_{i=1}^{n} \frac{X_i}{n}$

$$E[\bar{X}] = \theta \qquad Var(\bar{X}) = \frac{\sigma^2}{n}$$

Sample Variance

- σ^2 unknown in simulation
  - Hard to use $Var(\bar{X}) = \frac{\sigma^2}{n}$ to measure simulation variance
  - Thus we need to estimate σ^2
- Sample variance S^2:

$$S^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$$

  - n-1 instead of n is to provide unbiased estimator $E[S^2] = \sigma^2$

Estimate Error

- Sample mean $\bar{X}$ is a good estimator of θ, but has an error
  - How confidence we are sure that the sample mean is within an acceptable error?
- From central limit theorm:

$$\sqrt{n}\frac{(\bar{X}-\theta)}{\sigma} \sim N(0,1)$$

- It means that:

$$\sqrt{n}\frac{(\bar{X}-\theta)}{S} \sim N(0,1)$$

Confidence Interval

- R.v. Z$\sim$ N(0,1), for 0<α<1, define:
  - P(Z>z_α) = α
  - $z_{0.025}$ = 1.96
  - P(- $z_{0.025}$ <Z< $z_{0.025}$) =1-2α = 0.95

$$P(\bar{X} - z_{\alpha/2}\frac{S}{\sqrt{n}} < \theta < \bar{X} + z_{\alpha/2}\frac{S}{\sqrt{n}}) \approx 1 - \alpha$$

$$P(\bar{X} - 1.96\frac{S}{\sqrt{n}} < \theta < \bar{X} + 1.96\frac{S}{\sqrt{n}}) \approx 0.95$$

95% confidence interval (α = 0.05) of an estimate is:

$$(\bar{X} \pm 1.96 S/\sqrt{n})$$

When to stop a simulation?

- Repeatedly generate data (sample) until 100(1-α) percent confidence interval estimate of θ is less than I
  - Generate at least 100 data values.
  - Continue generate, until you generated k values such that $2z_{\alpha/2}S/\sqrt{k} < I$
  - The 100(1-α) percent confidence interval of estimate is

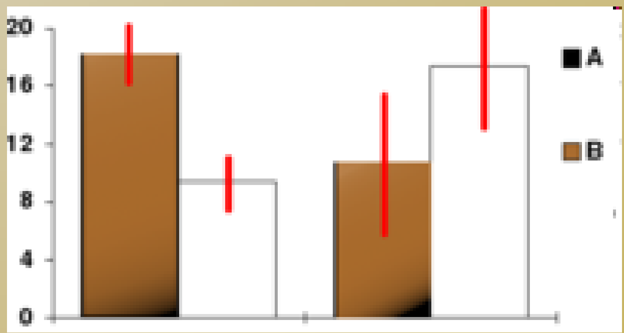
$$(\bar{X} - z_{\alpha/2}S/\sqrt{k}, \bar{X} + z_{\alpha/2}S/\sqrt{k})$$

Fix no. of simulation runs

- Suppose we only simulate 100 times
  - k=100
- What is the 95% confidence interval?

$$(\bar{X} - z_{\alpha/2} S/\sqrt{k}, \bar{X} + z_{\alpha/2} S/\sqrt{k})$$

$$(\bar{X} - 0.196 S/\sqrt{k}, \bar{X} + 0.196 S/\sqrt{k})$$



Example: Generating Expo. Distribution