LECTURE 11: Active Contour and Level Set for Medical Image Segmentation

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Outline

- Active Contour (Snake)
- Level Set
- Applications
Motivation

• Active contours and active surfaces are means of model-driven segmentation. Their use enforces closed and smooth boundaries for each segmentation **irrespective of the image content.**
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- **Model-driven (boundary) approaches:** Ideal object boundary are predicted. The boundary is assumed to be smooth and closed.
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Motivation
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Active Contours (Snake)

- First introduced in 1987 by Kass et al., and gained popularity since then.

- Represents an object boundary or some other salient image feature as a parametric curve.

- An energy functional $E$ is associated with the curve.

- The problem of finding object boundary is cast as an energy minimization problem.
A Snake is a parametric curve!

The course of the snake smoothly follows high intensity gradients if the gradients reliably reflect the object boundary. Otherwise, a smooth boundary is generated bridging regions of noisy data or missing gradients. Such an active contour is particularly well suited to segment an object instance in an image where the data are distorted by noise or artefacts.
Frameworks for Snakes

- A higher level process or a user initializes any curve close to the object boundary.
- The snake then starts deforming and moving towards the desired object boundary.
- In the end it completely “shrink-wraps” around the object.
Deformable Models

- Deformable models are curves or surfaces defined within an image domain that can move under the influence of internal forces,
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• Deformable models are curves or surfaces defined within an image domain that can move under the influence of internal forces, which are defined within the curve or surface itself, and external forces, which are computed from the image data.

The internal forces are designed to keep the model smooth during deformation.

The external forces are defined to move the model toward an object boundary or other desired features within an image.
Active Contour Modeling

- The contour is defined in the \((x, y)\) plane of an image as a parametric curve
  \[ \mathbf{v}(s) = (x(s), y(s)) \quad 0 \leq s \leq 1 \]

- Contour is said to possess an energy \((E_{\text{snake}})\) which is defined as the sum of the three energy terms.
  \[ E_{\text{snake}} = E_{\text{internal}} + E_{\text{external}} + E_{\text{constraint}} \]

- The energy terms are defined cleverly in a way such that the final position of the contour will have a minimum energy \((E_{\text{min}})\)

- Therefore our problem of detecting objects reduces to an energy minimization problem.

What are these energy terms which do the trick for us?
Internal Energy

- The smoothness energy at contour point $v(s)$ could be evaluated as

$$E_{in}(v(s)) = \alpha(s) \left| \frac{d v}{d s} \right|^2 + \beta(s) \left| \frac{d^2 v}{d^2 s} \right|^2$$

Elasticity/stretching  Stiffness/bending

Then, the interior energy (smoothness) of the whole snake $C = \{v(s) | s \in [0,1]\}$

$$E_{in} = \int_{0}^{1} E_{in}(v(s)) \, ds$$
Internal Energy

\[ C = (v_0, v_1, v_2, \ldots, v_{n-1}) \in \mathbb{R}^{2n} \]

\[ v_i = (x_i, y_i) \]

elastic energy (elasticity)

\[ \frac{dv}{ds} \approx v_{i+1} - v_i \]

bending energy (stiffness)

\[ \frac{d^2v}{ds^2} \approx (v_{i+1} - v_i) - (v_i - v_{i-1}) = v_{i+1} - 2v_i + v_{i-1} \]
Internal Energy

\[ C = (v_0, v_1, v_2, \ldots, v_{n-1}) \in \mathbb{R}^{2n} \]

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\]

\[
E_{in} = \sum_{i=0}^{n-1} \alpha |v_{i+1} - v_i|^2 + \beta |v_{i+1} - 2v_i + v_{i-1}|^2
\]

Min energy when curve minimizes length of contour. ....... ............... is smooth

Elasticity

Stiffness
External Energy

- The external energy describes **how well the curve matches the image data locally**
- Numerous forms can be used, attracting the curve toward different image features
External (Image) Energy

• Suppose we have an image \( I(x,y) \)
• Can compute image gradient \( \nabla I = (I_x, I_y) \) at any point
• Edge strength at pixel \( (x,y) \) is \( |\nabla I(x, y)| \)
• *External energy* of a contour point \( \mathbf{v}=(x,y) \) could be

\[
E_{ex}(\mathbf{v}) = -|\nabla I(\mathbf{v})|^2 = -|\nabla I(x, y)|^2
\]

**External energy** term for the whole snake is

\[
E_{ex} = \int_{0}^{1} E_{ex}(\mathbf{v}(s)) \, ds
\]

- continuous case \( C=\{\mathbf{v}(s) | s \in [0,1]\} \)
- discrete case \( C=\{\mathbf{v}_i | 0 \leq i < n\} \)

\[
E_{ex} = \sum_{i=0}^{n-1} E_{ex}(\mathbf{v}_i)
\]
Basic Elastic Snake

- The total energy of a basic elastic snake is

\[ E = \alpha \cdot \sum_{i=0}^{n-1} |v_{i+1} - v_i|^2 \]

- Continuous case:
  \[ E = \alpha \cdot \int_0^1 \left| \frac{dv}{ds} \right|^2 ds - \int_0^1 |\nabla I(v(s))|^2 ds \]

- Discrete case:
  \[ E = \alpha \cdot \sum_{i=0}^{n-1} |v_{i+1} - v_i|^2 - \sum_{i=0}^{n-1} |\nabla I(v_i)|^2 \]

(PS. bending energy can be added under elastic term)
Basic Elastic Snake

\[ C = (v_i \mid 0 \leq i < n) = (x_0, y_0, x_1, y_1, \ldots, x_{n-1}, y_{n-1}) \]

\[
E_{in} = \alpha \cdot \sum_{i=0}^{n-1} L_i^2 \quad \text{This can make a curve shrink} \\
= \alpha \cdot \sum_{i=0}^{n-1} (x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2
\]

\[
E_{ex} = -\sum_{i=0}^{n-1} |\nabla I(x_i, y_i)|^2
\]

\[
= -\sum_{i=0}^{n-1} |I_x(x_i, y_i)|^2 + |I_y(x_i, y_i)|^2
\]
Find Contour $C$ that minimizes $E(C)$

$$E(C) = \alpha \cdot \sum_{i=0}^{n-1} (x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 - \sum_{i=0}^{n-1} |I_x(x_i, y_i)|^2 + |I_y(x_i, y_i)|^2$$

Optimization problem for function of $2n$ variables
- can compute local minima via gradient descent
- more robust option: dynamic programming
Constraint Forces ($E_{\text{constraints}}$)

- Initial snake result can be nudged where it goes wrong, simply add extra external energy terms to
  
  - Pull nearby points toward cursor, or
    
    $$E_{\text{pull}} = - \sum_{i=0}^{n-1} \frac{r^2}{|v_i - p|^2}$$
  
  - Push nearby points away from cursor
    
    $$E_{\text{push}} = + \sum_{i=0}^{n-1} \frac{r^2}{|v_i - p|^2}$$
Ex: If Only External Force is Used

Red: initial contour
Green: final contour

Credit: Scot Acton
Gradient Descent

• Example: minimization of functions of 2 variables

- Negative gradient at point \((x, y)\) gives direction of the steepest descent towards lower values of function \(E\)

\[
- \nabla E = \begin{bmatrix}
- \frac{\partial E}{\partial x} \\
- \frac{\partial E}{\partial y}
\end{bmatrix}
\]
Gradient Descent

- Example: minimization of functions of 2 variables

\[ E(x, y) \]

\[ \left( x', y' \right) = \left( x, y \right) - \Delta t \cdot \begin{pmatrix} \frac{\partial E}{\partial x} \\ \frac{\partial E}{\partial y} \end{pmatrix} \]

Stop at a local minima where \( \nabla E = \vec{0} \)
Gradient Descent

- Example: minimization of functions of 2 variables

\[ E(x, y) \]

High sensitivity \textit{wrt.} the initialisation !!
Gradient Descent in Snakes

\[ E(x_0, \ldots, x_{n-1}, y_0, \ldots, y_{n-1}) = -\sum_{i=0}^{n-1} |I_x(x_i, y_i)|^2 + |I_y(x_i, y_i)|^2 \]

\[ + \alpha \cdot \sum_{i=0}^{n-1} (x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 \]

Here, energy is a function of 2n variables.

Simple elastic snake energy

Update equation for the whole snake

\[ C' = C - \nabla E \cdot \Delta t \]

\[ \begin{pmatrix} x'_0 \\ y'_0 \\ \vdots \\ x'_{n-1} \\ y'_{n-1} \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ \vdots \\ x_{n-1} \\ y_{n-1} \end{pmatrix} - \begin{pmatrix} \frac{\partial E}{\partial x_0} \\ \frac{\partial E}{\partial y_0} \\ \vdots \\ \frac{\partial E}{\partial x_{n-1}} \\ \frac{\partial E}{\partial y_{n-1}} \end{pmatrix} \cdot \Delta t \]
Dynamic Programming for Snakes

• Please Read
  – Interactive Segmentation with Intelligent Scissors by E. Mortensen and W. Barrett,
  – Using Dynamic Programming for Solving variational Problems in Vision by AA. Amini et al, where authors used dynamic programming for image segmentation tasks.
Ex: Corpus Collasum
Problems with Snakes

- Depends on number and spacing of control points
- Snake may over-smooth the boundary
- Initialization is crucial
- Not trivial to prevent curve self-intersecting
- May not follow topological changes of objects
Level Sets

• A limitation of active contours based on parametric curves of the form $f(s)$ (snakes, b-snakes,...) is that it is challenging to change the topology of the curve as it evolves.
• If the shape changes dramatically, curve reparameterization may also be required.
• An alternative representation for such closed contours is to use **level sets (LS).**
  – LS evolve to fit and track objects of interest by modifying the underlying embedding function instead of curve function $f(s)$
Image Segmentation with Level Sets

- Contour evolution (Sethian and Osher, 1988)
- Level sets for closed contours
  - Zero-crossing(s) of a characteristic function define the curve
  - Fit and track objects of interest by modifying the underlying embedding function $\phi(x, y)$ instead of the curve $f(s)$
  - Efficient algorithm
    - A small strip around the locations of the current zero-crossing needs to be updated at each step
Moving Interfaces

• 2D Moving Curves
• 3D Moving Surfaces

Ex:
  – Interfaces between water and oil
  – Propagating front of bush fire
  – Deformable elastic solid
Evolving Curves and Surfaces

- Propagate curve according to speed function $v = F_n$
- $F$ depends on space, time, and the curve itself
- Surfaces in three dimensions

Only velocity component normal to surface is important!
Describe curve as Level Sets of a Function

\[ \phi(x, y) = x^2 + y^2 - 1 = 0 \]

Isocontour is the unit circle (implicit representation.)
Describe curve as Level Sets of a Function

\[ \phi(x, y) = x^2 + y^2 - 1 = 0 \]

A few isocontours of two dimensional function (circle)
Along with some representative normals.

**GRADIENT:**

\[ \nabla \phi = \left( \frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y} \right) \]
Describe curve as Level Sets of a Function

Then, unit normal (outward) is

$$\hat{N} = \frac{\nabla \phi}{|\nabla \phi|}$$
Describe curve as Level Sets of a Function

Then, unit normal (outward) is

\[
\hat{N} = \frac{\nabla \phi}{|\nabla \phi|}
\]

On Cartesian grid, we need to approximate this equation (ex. Finite difference techniques):

\[
\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_i}{\Delta x}
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**Mean curvature** of the interface is defined as the divergence of the normal

\[ \vec{N} = (n_1, n_2) \]

\[ \kappa = \nabla \cdot \vec{N} = \frac{\partial n_1}{\partial x} + \frac{\partial n_2}{\partial y} = \nabla \cdot \left( \frac{\nabla \phi}{|\nabla \phi|} \right) \]
Describe curve as Level Sets of a Function

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Mean curvature of the interface is defined as the divergence of the normal \( \vec{N} = (n_1, n_2) \)

\[ \kappa = \nabla \cdot \vec{N} = \frac{\partial n_1}{\partial x} + \frac{\partial n_2}{\partial y} = \nabla \cdot \left( \frac{\nabla \phi}{|\nabla \phi|} \right) \]
Variational Formulations and LS

• Transition from Active Contours:
  – contour $v(t) \rightarrow$ front $\gamma(t)$
  – contour energy $\rightarrow$ forces $F_A$, $F_C$
  – image energy $\rightarrow$ speed function $k_I$

• Level set:
  – The level set $c_0$ at time $t$ of a function $\psi(x,y,t)$
    is the set of arguments $\{ (x,y) , \psi(x,y,t) = c_0 \}$
  – Idea: define a function $\psi(x,y,t)$ so that at any time,
    $\gamma(t) = \{ (x,y) , \psi(x,y,t) = 0 \}$

  • there are many such $\psi$
  • $\psi$ has many other level sets, more or less parallel to $\gamma$
  • only $\gamma$ has a meaning for segmentation, not any other level set of $\psi$
Usual choice for $\psi$: signed distance to the front $\gamma(0)$

$$\psi(x,y,0) = \begin{cases} 
- d(x,y, \gamma) & \text{if} \ (x,y) \text{ inside the front} \\
0 & \text{" on "} \\
d(x,y, \gamma) & \text{" outside "}
\end{cases}$$
Front Propagation

\[ \frac{\partial \psi}{\partial t} + \hat{k}_I \cdot (F_A + F_G(\kappa)) \cdot \|\nabla \psi\| = 0 \]

- **Product of influences**
- **Spatial derivative of \( \psi \)**
- **Constant “force”** (balloon pressure)
- **Smoothing “force”** depending on the local curvature \( \kappa \) (contour influence)
- **Extension of the speed function \( k_I \)** (image influence)
- \( \psi(x,y,t+1) - \psi(x,y,t) \)
- **Link between spatial and temporal derivatives, but not the same type of motion as contours!**

\( \kappa = \text{div} \left( \frac{\nabla \psi}{\|\nabla \psi\|} \right) \)
Front Propagation

- Speed function:
  - $k_I$ is meant to stop the front on the object’s boundaries
  - similar to image energy: $k_I(x,y) = 1 / (1 + ||\nabla I(x,y)||)$
  - only makes sense for the front (level set 0)
  - yet, same equation for all level sets
    - extend $k_I$ to all level sets, defining $\hat{k}_I$

- possible extension:
  - $\hat{k}_I(x,y) = k_I(x',y')$
    where $(x',y')$ is the point in the front closest to $(x,y)$
  - (such a $\hat{k}_I(x,y)$ depends on the front location)
Reconstruction of Surfaces from Unorganized Data Points

(a) data points  (b) initial guess  (c) final reconstruction

Reconstruction of a rat brain from data of MRI slices
Ultrasound image segmentation.

*Chunming Li et al.*
LS Evolution without reinitialization: a new variational formulation.
Vein Segmentation with Level Set
Spinal Cord Quantification - MRI

- Atrophy (Multiple-Sclerosis) is generally assessed by measuring the cross-sectional areas at specific levels (typically C2–C5) along the cervical cord.

- Spinal cord under analysis can be characterized by a bright structure against a dark background.

- Segmentation is necessary for accurate and automatic quantification.
Spinal Cord Segmentation in MRI

Surface evolution during the segmentation process of spinal cord from the MRI image (the number in the left corner of each image represents the number of elapsed iterations).

\[
\frac{\partial \phi}{\partial t} = \left[ g \cdot \text{div} \left( \frac{\nabla \phi}{|\nabla \phi|} \right) |\nabla \phi| + \text{sign}(\nabla g \nabla \phi) (\nabla g \nabla \phi) + v \nabla g \right] \|\nabla \phi\|.
\]

Selective contrast

Credit: Dougherty, MIP.
Cyst Segmentation from Breast US Images

Contour extraction of cyst form ultrasound breast image via merging multiple initial level sets. Images courtesy of Yezzi, Georgia Institute of Technology.
Shape Constraints for LV Segmentation – Cardiac MRI
(Yuanquan Wang, et al, Shape Analysis in Medical Image Analysis)

- Extensive techniques available for cardiac imaging provide qualitative and quantitative information about the morphology and function of the heart and great vessels
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Shape Constraints for LV Segmentation – Cardiac MRI
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• Extensive techniques available for cardiac imaging provide qualitative and quantitative information about the morphology and function of the heart and great vessels
• Many clinically established diagnosis indices such as wall thickness, myocardial motion, ejection fraction, and circumferential shortening of myocardial fibers are evaluated by the segmentation results of MRIs.
• In clinical practice, the LV segmentation task is often performed manually by an experienced clinician. Manual segmentation of the LV, however, is tedious, time consuming, subjective and irreproducible.
A major difficulty in segmentation of the cardiac MR images is the intensity inhomogeneity due to the radio-frequency coils or acquisition sequences. The myocardium and surrounding tissues such as the liver have almost the same intensity profile, leading to low contrast between them.
Endocardium Segmentation - MRI

(a) Failed active contour segmentations without the circle-shape constraint. 
(b) Succeeded segmentations with the circle-shape constraint

\[ E(C) = \int_0^1 \left( \frac{1}{2} \left( \alpha |C'(s)|^2 + \beta |C''(s)|^2 \right) + g(C(s)) + \frac{E_{Cir}(C(s))}{GVF} \right) ds. \]

\[ E_{cir}(x, y) = \frac{\lambda}{2} \int_0^1 (R(s) - \overline{R})^2 ds, \]
Epicardium Segmentation - MRI

**a** Epicardium extraction using new external force. **b** Comparison of segmentation results with and without shape (similarity) energy

\[ E(C) = \int_{0}^{1} \left( \frac{1}{2} \left( \alpha \left| C'(s) \right|^2 + \beta \left| C''(s) \right|^2 \right) + \frac{g(C(s))}{GVC} + \frac{E_{siml}(C(s))}{similarity\ constraint} \right) ds, \]
Shape Similarity Constraint

- There would be spurious edges on the myocardium, and the contrast between myocardium and surrounding structures would be low. Authors employ the endocardium result as a priori shape and construct a new shape-similarity based constraint given by

\[ E_{\text{simi}} = \frac{\rho}{2} \int_0^1 \left( (R(s) - \bar{R}) - (r(s) - \bar{r}) \right)^2 ds. \]

\( R(s) - R \) measures the deviation of the snake contour for epicardium from a circle with radius R at snaxel s. \( r(s) \)’s are for endocardium.
LS Evolution with Region Competition (Ho et al., ICPR 2003)

- Good initialization → one major problem in snakes
- Shape constraint based LS is good, but not easy to construct shape constraint
- Missing/fuzzy boundary -> leakage due to constant propagation force
  - Two adjacent regions compete for the common boundary
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• Tumors vary in shape, texture, size, and intensity
• T1-MRI is used for detailed neuroanatomy, but not good for precisely distinguishing tumor regions
• T2-MRI is good for tumor and edema identification, but often it is difficult to obtain high resolution
• Post-contrast T1-weighted MRI is more suitable for tumor segmentation
Without (left) and with (right) contrast agent, T1-weighted MRI
LS Evolution with Region Competition (Ho et al., ICPR 2003)

• New formula modulates the propagation term using image forces to change the direction of propagation, so that the snake shrinks when the boundary encloses parts of the background (B), and grows when the boundary is inside the tumor region (A):

\[
\frac{\partial \phi}{\partial t} = \alpha(P(A) - P(B))|\nabla \phi| \\
+ c_{\text{MCF}} \nabla \cdot \left( \frac{\nabla \phi}{|\nabla \phi|} \right) |\nabla \phi| \\
+ c_{\text{sm}} \nabla^2 \phi
\]
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Region competition

Controls strength of Smoothing (on active contour)
LS Evolution with Region Competition (Ho et al., ICPR 2003)

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\]

Region competition
Controls strength of Smoothing (on active contour)
Controls strength of Smoothing (on LS contour)
LS Evolution with Region Competition (Ho et al., ICPR 2003)

**Tumor probability map** (orange: highly likely tumor regions) is obtained after registering pre- and post-contrast T1 MR images.
LS Evolution with Region Competition (Ho et al., ICPR 2003)

Tumor probability map (orange: highly likely tumor regions) is obtained after registering pre- and post-contrast T1 MR images.

This map is used to initialize proposed LS segmentation method.
Level Set Segmentation in Slicer

• Following examples (slides) are from NA-MIC
Minimal curvature
Upwind Vector
Slide Credits and References

- **Credits to:** M.Brady and R.Szelisky, Bagci’s CV Course 2015 Fall.
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- Osher and Paragios (2003), Paragios, Faugeras, Chan et al. (2005), Paragios and Sgallari (2009)
- G. Strang, Lecture Notes, MIT.
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- Sethian, JA. Fast Marching. PNAS 1996.
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- Lim, Bagci, and Li. IEEE TBME 2013 [Willmore Flow and Level Set]
- K.D. Toennies, Guide to Medical Image Analysis,