Computer Science Foundation Exam

August 8, 2014

Section II B

DISCRETE STRUCTURES

NO books, notes, or calculators may be used,
and you must work entirely on your own.

SOLUTION

Question	Max Pts	Category	Passing	Score
1	15	CTG (Counting)	10	
2	15	PRB (Probability)	10	
3	10	PRF (Functions)	6	
4	10	PRF (Relations)	6	
ALL	50		32	

You must do all 4 problems in this section of the exam.

Problems will be graded based on the completeness of the solution steps and <u>not</u> graded based on the answer alone. Credit cannot be given unless all work is shown and is readable. Be complete, yet concise, and above all <u>be neat</u>.

1) (15 pts) CTG (Counting)

Please leave your answers in factorials, permutations, combinations and powers. Do not calculate out the actual numerical value for any of the questions.

There are twenty tasks that need to be completed. Each of the following questions deals with this scenario.

(a) (3 pts) In how many different orders can the tasks be completed by a single individual, if each task is unique?

Since the tasks are unique, there are 20! different orders in which they can be completed. (Grading: 3 pts all or nothing, no explanation needed.)

(b) (5 pts) Assume that each task is unique and that there are five distinguishable workers who must collectively complete the tasks, with no restrictions on who can complete which tasks. In how many ways can the tasks be assigned to the workers?

The number of different assignments can be counted by seeing that we can assign each task to any of 5 workers. Thus, the product principle can be used to determine that there are 5^{20} possible ways to assign the tasks to the workers. (Grading: 5 pts for correct answer, 3 pts for switching the exponent and base, 0 pts otherwise.)

(c) (7 pts) In this part, assume that the tasks are indistinguishable and the workers are distinguishable. Furthermore, assume that each of the five workers must be assigned to at least one task. In how many different ways can the tasks be assigned to the workers? (Note: Two combinations count separately if at least one specific worker, say Dave, has a different *number* of tasks to do in the two combinations.)

This problem set up is combinations with repetition, where the workers are the types of different items and the tasks are the number of total items. Let the number of tasks workers 1 through 5 have be a, b, c, d, and e, respectively. Then, we must find the number of positive integer solutions (since each worker must have at least one task) to the equation

$a + b + c + d + e = 20$

Given each worker 1 task so that we have 15 left to assign, so we're looking at finding the number of non-negative integer solutions to the equation $a' + b' + c' + d' + e' = 15$. (Note: $a' = a + 1$, and so on.) Using the combinations with repetition formula, we get a total of $\binom{15+5-1}{5-1} = \binom{19}{4}$ number of different ways to assign the tasks.

Grading: 3 pts for recognizing combinations with repetition, 2 pts for subtracting out 5, 2 pts for applying the formula correctly from there.

2) (15 pts) PRB (Probability)

(a) (5 pts) John is rolling two fair six-sided dice with the following number of dots on each side: 1, 3, 5, 7, 9 and 11. What is the probability that on a single roll, the number of dots showing on the two dice is less than 12? Express your answer as a fraction in lowest terms.

An exhaustive search of the 36 possible rolls yields the following ordered pairs of dice rolls that have a sum less than 12: (1, 1), (1, 3), (1, 5), (1, 7), (1, 9), (3, 1), (3, 3), (3, 5), (3, 7), (5, 1), (5, 3), (5, 5), (7, 1), (7, 3), and (9, 1), for a total of 15 rolls out of 36 which sum to less than 12. Thus, the desired probability is $\frac{15}{36} = \frac{5}{12}$.

Grading: 2 pts for correct sample space, 2 pts for counting successes, 1 pt for final answer

(b) (10 pts) The chance of a computer system with a crytographic weakness being broken is 95% while the chance of a computer system without a cryptography weakness being broken is 2%. In a large study, it was found that .3% of computer systems had cryptographic weaknesses. CompsRUs had its computer system broken. What is the probability that CompsRUs had a cryptographic weakness? Since the division is difficult to do by hand, leave your answer as the quotient of two values. (Please do work out these two values by hand.)

This is similar to a Bayes' Law problem. We can ascertain that 99.7% of companies don't have a cryptographic weakness. The probability that some company has a break in is:

.003 x .95 + .997 x .02 = 2.279% (6 pts)

The probability that a company has a cryptographic weakness AND it's broken into is:

.003 x .95 = .00285 (3 pts)

Thus, the desired probability is $\frac{.00285}{.02279} \sim 12.5\%$. (1 pt)

3) (10 pts) PRF (Functions)

Let $f(x) = 3x + 5$. Let $f^n(x)$ to be the function f composed with itself n times. (For example, $f^3(x) = f(f(f(x)))$.) Determine $f^4(x)$.

Let $g(x) = f(f(x)$. We desire to find $g(g(x))$.

$g(x) = f(f(x) = f(3x + 5) = 3(3x + 5) + 5 = 9x + 15 + 5 = 9x + 20$ (5 pts to get here)

$g(g(x) = g(9x + 20) = 9(9x + 20) + 20 = 81x + 180 + 20 = 81x + 200$ (5 more pts for here)

Grading (partial credit): Allocate the 5 points consistently as you see fit. They should be for each step of the process.

4) (10 pts) PRF (Relations)

Consider the following relation R, between UCF students:

R = { (x, y) | there exists some instructor z such that both x and y have taken classes from z }

We define a UCF student to be anyone who has taken at least one class from an instructor at UCF.

With proof, determine which of the following properties R satisfies:

(a) reflexive, (b) irreflexive, (c) symmetric, (d) anti-symmetric, and (e) transitive.

(a) Yes, since any person x who is a UCF student has been in at least one class, they have been in at least one class with themselves.

(b) No, since there is at least one ordered pair, (John, John) that is part of the relation. (John is a specific UCF student.)

(c) Yes, since if x and y have been in one class together, so have y and x.

(d) No, Bob and Jane took a UCF class together and Jane and Bob took a UCF class together and Jane and Bob are different people, so the relation can not be anti-symmetric.

(e) No, Consider a situation where Bob and Jane took Zoology together and Jane and Amelia took Calculus together, but Bob never took a class with Amelia. (This type of situation is typical…since we are free to choose different classes.)

Grading – 1 pt for each result and 1 pt for each explantion.