Computer Science Foundation Exam

August 13, 2010

Section I A

COMPUTER SCIENCE

NO books, notes, or calculators may be used, and you must work entirely on your own.

Name: ________________________________________________________________

PID: ________________________________________________________________

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You must do all 5 problems in this section of the exam.

Problems will be graded based on the completeness of the solution steps and not graded based on the answer alone. Credit cannot be given unless all work is shown and is readable. Be complete, yet concise, and above all be neat.
1) (10 points) **Recursion** Write a **recursive** function that sums the *odd* digits of an integer. For example, if your function takes in the number 8135267, it should return 16 because 1+3+5+7=16. (Note that 8, 2 and 6 were ignored since they are even.) You may assume that the function will only be passed non-negative integers.

```c
int sumOddDigits(int number)
{
    if (number == 0)
        return 0;
    if (number%2 == 1)
        return number % 10 + sumOddDigits(number / 10);
    else
        return sumOddDigits(number / 10);
}
```

**Grading Criteria:**
There are many ways to approach this problem. Be reasonable when grading.
Base case – 3 points
Differentiating odd/even units digit cases – 3 pts
Adding number%10 only in odd case – 2 pts
Proper recursive calls in both cases – 2 pts
2) (10 points) **Summations**

a) Determine a simplified closed-form solution for the following summation in terms of $n$:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} (5i + 3j)$$

b) Evaluate the following summation: $$\sum_{i=1}^{20} 3i$$

**Correct Answers:**

a) 

$$\sum_{i=1}^{n} \sum_{j=1}^{n} (5i + 3j) = \sum_{i=1}^{n} \left(5i + \sum_{j=1}^{n} 3j\right) = \sum_{i=1}^{n} \left(5in + \frac{3n(n+1)}{2}\right) = \sum_{i=1}^{n} 5in + \sum_{i=1}^{n} \frac{3n(n+1)}{2}$$

$$= 5n \frac{n(n+1)}{2} + 3n \frac{n(n+1)}{2} = 8 \frac{n^2(n+1)}{2} = 4n^2(n+1)$$

b) 

$$\sum_{i=1}^{20} 3i = 3 \left(\sum_{i=1}^{20} i - \sum_{i=1}^{10} i\right) = 3 \left(\frac{20 \cdot 21}{2} - \frac{10 \cdot 11}{2}\right) = 3(210 - 55) = 3 \cdot 155 = 465$$

**Grading Criteria:**

a)  
Properly dealing with the inner summation – 2 points  
Properly dealing with the outer summation – 2 points  
Simplifying the resulting closed form – 1 point

b)  
Properly splitting the summation into two parts – 2 points  
Evaluating the two resulting summations properly – 2 points  
Correctly determining the final answer – 1 point
3) (10 points) **Stack Applications** Evaluate the postfix expression below using a stack. Show the contents of the stack at the indicated points 1, 2 and 3 in the postfix expression.

\[
\begin{array}{cccccc}
8 & 6 & 8 & * & 9 & 5 & - & / & 1 & 4 & + & 2 & * & - & / \\
\end{array}
\]

**Correct Answers:**

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<td>48</td>
<td>12</td>
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<tr>
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1 | 2 | 3

Resulting answer:

4

**Grading Criteria:**
- Stack 1 – 2 points
- Stack 2 – 3 points
- Stack 3 – 3 points
- Final answer – 2 points
4) (10 points) **Binary Heaps** Consider the following binary min-heap:

![Binary Min-Heap Diagram]

**a)** Show the state of the heap after inserting the value 1. Show both the state before any percolations and the final state.

**b)** Show the result of deleting the minimum value from the original heap (i.e. ignore part a when answering this part). Show both the state before any percolations and the final state.

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**Correct Answers:**

**a)**

![Inserting 1 Diagram]

**b)**

![Deleting Min Diagram]

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**Grading Criteria:**

- 5 points per part
- 2 points for a proper “before” state
- 3 points for correctly handling the percolation
5) (10 points) **Binary Tree Traversals**

Give the preorder, inorder, and postorder traversals of the binary tree shown above.

**Correct Answers:**

**Preorder:**
8, 3, 2, 1, 6, 4, 5, 7, 10, 9, 11

**Inorder:**
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

**Postorder:**
1, 2, 5, 4, 7, 6, 3, 9, 11, 10, 8

**Why is the tree depicted above not a valid AVL tree?**
The height of the sub-trees rooted at nodes 3 and 10 differ in height by more than 1 level

**Grading Criteria:**
3 points for each traversal
1 point for determining why it’s not an AVL tree