Computer Science Foundation Exam

May 7, 2010

Computer Science

Section 1A

Name: SOLUTION

	Max Pts	Type	Passing Threshold	Student Score
Q1	**10**	**DSN**	**7**	
Q2	**10**	**ANL**	**7**	
Q3	**10**	**ALG**	**7**	
Q4	**10**	**ALG**	**7**	
Q5	**10**	**ALG**	**7**	
Total	**50**		35	

You must do all 5 problems in this section of the exam.

Partial credit cannot be given unless all work is shown and is readable. Be complete, yet concise, and above all be neat. Do your rough work on the last page.

1) (10 points) **Recursion** Money in bank accounts grows at a rate known as the annual percentage yield, or APY. For example, if you have $1,000 in the bank with an APY of 1.10, then after one year, you'll have $1,100 in the bank. The APY is applied to the new balance every year, so if you leave $1,000 in the bank for two years, you'll end up with ($1,100 * 1.10) = $1,210. Write a **<u>recursive</u>** function to compute and return the amount of money in a bank account after a number of years. This function should take in the starting amount and the APY as `double`s, and the number of years as an `int`.

```
// Pre-condition: money > 0, 1 < apy < 2, 0 <= years <= 100
double bank(double money, double apy, int years)
{

     if (years <= 0) // 2 pts - okay if it says ==
          return money; // 2 pts
     else               // 1 pt
          return bank(money*apy, apy, years-1);
          // 1 pt return, 1 pt rec call, 1 pt for each param

}
```

2) (10 points) **Summations**

a) Determine a closed-form solution for the following sum in terms of n: $\sum_{j=n-10}^{n} 5jn$.

(You may assume that $n \geq 12$)

$$= \sum_{j=n-10}^{n} 5jn = 5n \sum_{j=n-10}^{n} j \qquad = 5n\left(\sum_{j=1}^{n} j - \sum_{j=1}^{n-11} j\right) \qquad = 5n\left(\frac{n(n+1)}{2} - \frac{(n-11)(n-10)}{2}\right)$$

$$= 5n\left(\frac{n^2 + n - (n^2 - 21n + 110)}{2}\right) = \frac{5n}{2}(n^2 + n - n^2 + 21n - 110) = \frac{5n(22n - 110)}{2} = 55n(n-5)$$

Grading: (6 pts) 1 pt for factoring out 5n, 2 pts for splitting the sum into two, 1 pt for evaluating each sum, 1 pt for simplifying

b) Determine a closed-form solution for the following sum in terms of n: $\sum_{i=1}^{n}\sum_{j=1}^{i}(2i)$

HINT: $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

$$= \sum_{i=1}^{n}\sum_{j=1}^{i}(2i) = 2\sum_{i=1}^{n} i^2 = \frac{2n(n+1)(2n+1)}{6} = \frac{n(n+1)(2n+1)}{3}$$

Grading: (4 pts total) 1 pt for factoring out 2, 1 pt for constant sum of i, 1 pt for plugging in formula for i^2, 1 pt for simplifying the answer

3) (10 points) **Stack Applications** Transform the following infix expression into its equivalent postfix expression using a stack. Show the contents of the stack at the indicated points 1, 2 and 3 in the infix expressions.

					1				**2**					**3**		
A	*****	**B**	**/**	**C**	**-**	**D**	**/**	**(**	**E**	*****	**F**	**+**	**G**	**)**	**+**	**H**

/

1

(
/
-

2

/
-

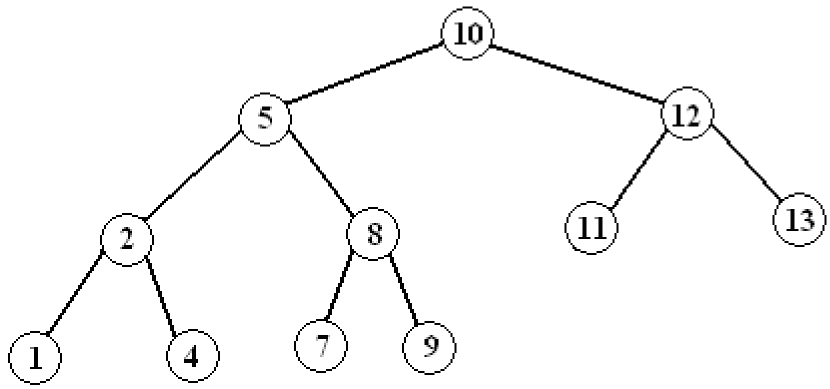
3

Resulting postfix expression:

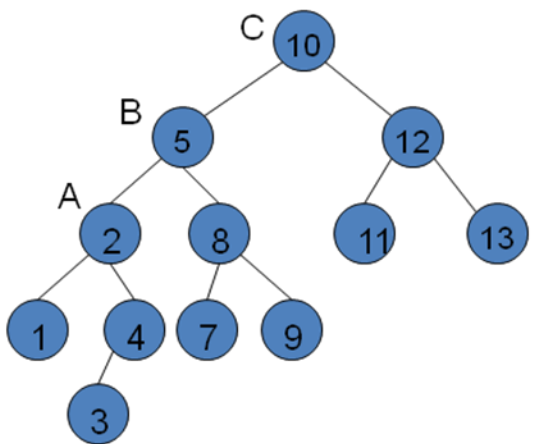
A	B	*	C	/	D	E	F	*	G	+	/	-	H	+					

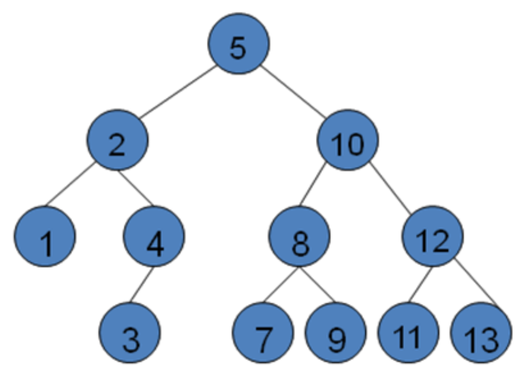
Grading: 1 pt off for each mistake (don't count the same mistake in both the stack and the expression twice), cap at 10.

4) (10 points) **AVL Trees** Consider the following AVL Tree:



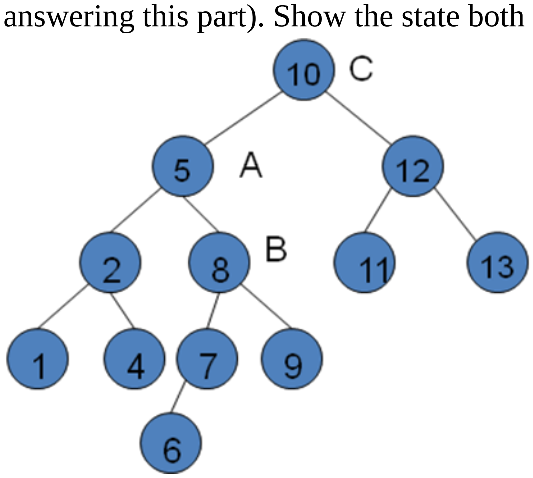
a) Show the state of the AVL after inserting the value 3. Show the state both before and after any necessary rotations.

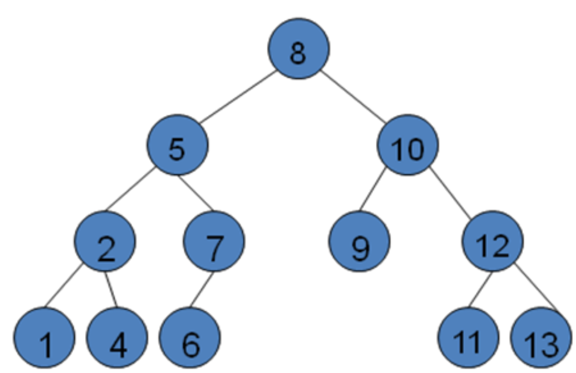




Grading: (5 pts total) 1 pt for 5 at root, 1 pt for 2 left, 1 pt 10 right, 2 pts for rest

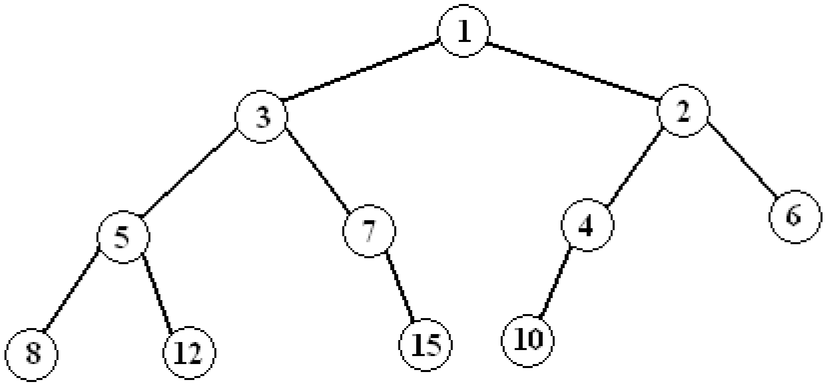
b) Show the result of inserting the value 6 into the original AVL tree (i.e. ignore part a when answering this part). Show the state both before and after any necessary rotations.





Grading: (5 pts total) 1 pt 8 root, 1 pt 5 left, 1 pt 10 right, 2 pts rest

5) (9 points) **Binary Tree Traversals**



Give the preorder, inorder, and postorder traversals of the binary tree shown above.

Preorder:

<u>1, 3, 5, 8, 12, 7, 15, 2, 4, 10, 6</u>

Inorder:

<u>8, 5, 12, 3, 7, 15, 1, 10, 4, 2, 6</u>

Postorder:

<u>8, 12, 5, 15, 7, 3, 10, 4, 6, 2, 1</u>

<u>Why is the tree depicted above not a valid binary heap?</u>
It is not a valid heap because the values are NOT filled in for each level, from left to right. In particular, 7 has no left child.

Grading: 3 pts for each traversal, 2 points off if they switch traversals, otherwise, eyeball how "close" it is, giving 2 for ones that are mostly correct, and 1 for ones that have 3-5 numbers in the correct slots

Last question: just 1 point