# Computer Science Foundation Exam

## December 18, 2009

## Section II A

## DISCRETE STRUCTURES SOLUTIONS

**NO books, notes, or calculators may be used, and you must work entirely on your own.**

**Name: ______________________________**

**PID: ______________________________**

**In this section of the exam, there are three (3) problems.**
**You must do ALL of them.**
**They count for 40% of the Discrete Structures exam grade.**
**Show the steps of your work carefully.**

**Problems will be graded based on the completeness of the solution steps and not graded based on the answer alone.**

**Credit cannot be given when your results are unreadable.**

| Question # | Max Pts | Category | Passing | Score |
|---|---|---|---|---|
| 1 | 15 | PRF (Induction) | 10 | |
| 2 | 15 | PRF (Sets) | 10 | |
| 3 | 10 | PRF (Logic) | 6 | |
| ALL | 40 | --- | 26 | |

**1)** (15 pts) PRF **(**Induction)

Using induction to prove that to prove that $\sum_{i=1}^{n}(3i+1)=\frac{n(3n+5)}{2}$ for all n ≥ 1.

**Base case: n =1,** $(3+1)=\frac{(3+5)}{2}$**, which is true. (2 pts)**

**Inductive hypothesis: Assume for an arbitrary non-negative integer n = k that** $\sum_{i=1}^{k}(3i+1)=\frac{k(3k+5)}{2}$**. (2 pts)**

**Inductive step: Prove that for n = k+1 that** $\sum_{i=1}^{k+1}(3i+1)=\frac{(k+1)(3(k+1)+5)}{2}$**. (2 pts)**

$\sum_{i=1}^{k+1}(3i+1)=\sum_{i=1}^{k}(3i+1)+[3(k+1)+1]$ **(2 pts)**

$=\frac{k(3k+5)}{2}+[3(k+1)+1]$ **, using inductive hypositheis (2 pts)**

$=\frac{k(3k+5)}{2}+3k+4$

$=\frac{3k^2+5k+6k+8}{2}$ **(3pts)**

$=\frac{3k^2+11k+8}{2}$

$=\frac{(k+1)(3k+8)}{2}$

$=\frac{(k+1)(3(k+1)+5)}{2}$ **which completes the proof. (2 pts)**

**2)** (15 pts) PRF **(**Sets)

Prove the following for arbitrarily chosen sets $A$, $B$ and $C$:

$$(B-A)\cup(C-A)=(B\cup C)-A$$

$(B-A)\cup(C-A)$
$=\{x|x\in(B-A)\vee x\in(C-A)\}$, by definition of union. **(3pts)**
$=\{x|(x\in B\wedge x\notin A)\vee(x\in C\wedge x\notin A)\}$, by definition of difference **(3pts)**
$=\{x|(x\in B\vee x\in C)\wedge x\notin A\}$, by Distributive Law for logical equivalence **(3pts)**
$=\{x|(x\in(B\cup C)\wedge x\notin A\}$, by definition of union **(3pts)**
$=(B\cup C)-A$, by definition of difference **(3pt)**

**Note to the grader: There are other possible ways in which to prove that these sets are equal. One way would be to show that the first side is a subset of the second, and vice versa. If this approach is taken give 9 points to the first half of the proof and 6 points to the second half.**

**3)** (10 pts) (PRF) Logic

Use the Laws of Logic and Rules of Inference to justify the following argument:

$(\neg p \vee \neg q) \rightarrow (s \wedge r)$
$s \rightarrow t$
$\neg t$
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$\therefore p$

Please name the Law of Logic or Rule of Inference used in each step of your proof.

| | |
|---|---|
| 1. $\neg t$ | Premise |
| 2. $s \rightarrow t$ | Premise |
| 3. $\neg s$ | Modus Tollens with (1) and (2) |
| 4. $\neg s \vee \neg r$ | Disjunctive Amplification using (3) |
| 5. $\neg(s \wedge r)$ | De Morgan's Laws |
| 6. $(\neg p \vee \neg q) \rightarrow (s \wedge r)$ | Premise |
| 7. $\neg(\neg p \vee \neg q)$ | Modus tollens with (5) and (6) |
| 8. $(\neg(\neg p)) \wedge (\neg(\neg q))$ | De Morgan's Laws |
| 9. $p \wedge q$ | Double negation Laws |
| 10. $p$ | Rule of Conunctive Simplification of (9). |

**Grading: 1 point per step. If all of the reasons are missing, just take off 3 points.**