# Computer Science Foundation Exam

**December 18, 2009**

# Computer Science

# Section 1A

**SOLUTION**

**PID:** _____

| | Max Pts | Type | Passing Threshold | Student Score |
|---|---|---|---|---|
| **Q1** | **11** | **DSN** | **8** | |
| **Q2** | **10** | **ANL** | **7** | |
| **Q3** | **10** | **ALG** | **7** | |
| **Q4** | **10** | **ALG** | **7** | |
| **Q5** | **9** | **ALG** | **6** | |
| Total | **50** | | 35 | |

**You must do all 5 problems in this section of the exam.**

**Partial credit cannot be given unless all work is shown and is readable. Be complete, yet concise, and above all be neat. Do your rough work on the last page.**

**1**) (11 points) **Recursion** Write a **recursive** function that sorts its input array values of length n as follows:

1) Finds the maximum item in the array in indexes 0 through n-1.
2) Swaps this element with the one stored in index n-1.
3) Recursively sorts the array values of length n-1.

```
void rec_sort(int values[], int n)
{

    if (n < 2) return; // 2 pts  - 0 also valid base case.

    int maxIndex = 0; // 1 pt
    int i;

    for (i=1; i<n; i++) // 1 pts
        if (values[i] > values[maxIndex]) // 1 pts
            maxIndex = i; // 1 pt

    int temp = values[n-1]; // 1 pt
    values[n-1] = values[maxIndex]; // 1 pt
    values[maxIndex] = temp; // 1 pt

    // 2 pts
    sort(values, n-1);

}
```

**2)** (10 points) **Summations**

**a)** Determine a closed-form solution for the following sum in terms of n: $\sum_{k=5}^{2n}(3k-2)$.

$$\sum_{k=5}^{2n}(3k-2)=\sum_{k=1}^{2n}(3k-2)-(1+4+7+10)=\frac{3(2n)(2n+1)}{2}-2(2n)-22$$
$$=3n(2n+1)-4n-22=6n^2+3n-4n-22=6n^2-n-22$$

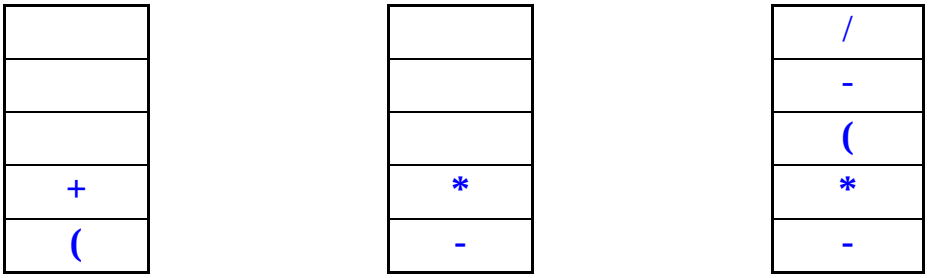
**Grading: 1 pt for small sum, 1 pt for large sum, 1 pt for final answer**

**b)** Determine a closed-form solution for the following sum in terms of n: $\sum_{i=0}^{n}\left(2\sum_{j=n+1}^{3n}(i+j)\right)$

$$\sum_{i=0}^{n}\left(2\sum_{j=n+1}^{3n}(i+j)\right)=\sum_{i=0}^{n}\left(2(\sum_{j=n+1}^{3n}i)+2(\sum_{j=n+1}^{3n}j)\right)$$
$$=\sum_{i=0}^{n}\left(2i(2n)+2(\sum_{j=1}^{3n}j-\sum_{j=1}^{n}j)\right)$$
$$=\sum_{i=0}^{n}\left(4in+2(\frac{3n(3n+1)}{2}-\frac{n(n+1)}{2})\right)$$ **(1 pt for each component, 3 pts total)**
$$=\sum_{i=0}^{n}\left(4in+3n(3n+1)-n(n+1)\right)$$
$$=\sum_{i=0}^{n}\left(4in+n[3(3n+1)-(n+1)]\right)$$
$$=\sum_{i=0}^{n}\left(4in+n[9n+3-n-1]\right)$$
$$=\sum_{i=0}^{n}\left(4in+n[9n+3-n-1]\right)$$
$$=\sum_{i=0}^{n}\left(4in+n(8n+2)\right)$$
$$=\frac{4n(n+1)n}{2}+n(8n+2)(n+1)$$ **(1 pt for other sum, 2 pt for simplify)**
$$=2n^2(n+1)+n(8n+2)(n+1)$$
$$=2n(n+1)[n+(4n+1)]$$
$$=2n(n+1)(5n+1)=10n^3+12n^2+2n$$ **(1 pt final answer, either form)**

**3)** (10 points) **Stack Applications** Transform the following infix expression into its equivalent postfix expression using a stack. Show the contents of the stack at the indicated points 1, 2 and 3 in the infix expressions.

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | 1 | | | | | | 2 | | | | | | 3 |
| **(** | **(** | **A** | **/** | **B** | **)** | **+** | **C** | **)** | **-** | **D** | ***** | **(** | **E** | **-** | **F** | **/** | **G** | **)** |



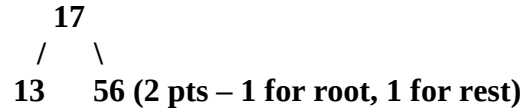

**Grading: 2 pts for each stack, 4 pts for the whole expression (partial credit allowed.)**

Resulting postfix expression:

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | B | / | C | + | D | E | F | G | / | - | * | - | | | | | | |

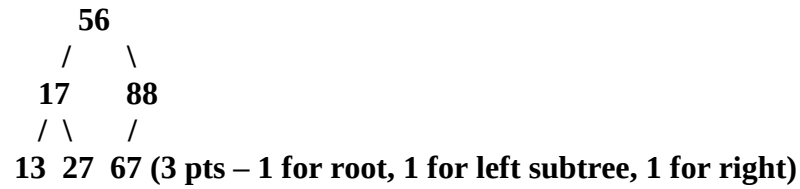
**4)** (10 points) **AVL Trees** Draw the resulting AVL tree after inserting the following items (in this order) into an initially empty AVL tree: 56, 17, 13, 88, 27, 67, 35, 20, 5, 28. Show the tree after each step that requires a rebalance. (There are three of these steps.)

**After first rebalance:**



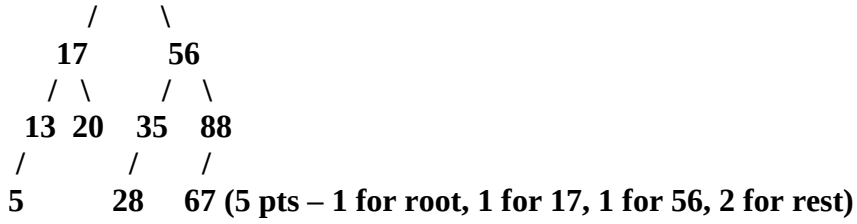

**(2 pts – 1 for root, 1 for rest)**

**After second rebalance:**



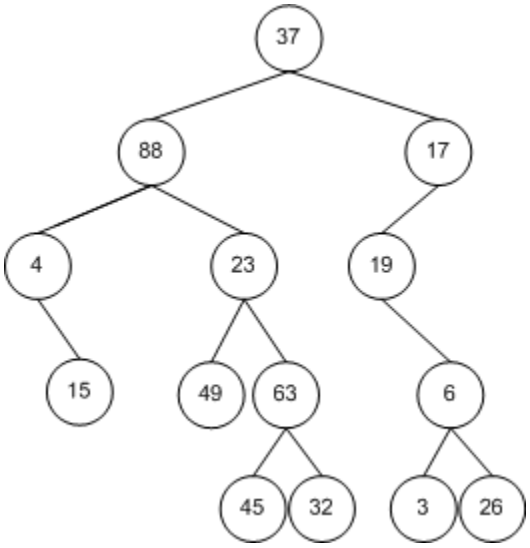

**(3 pts – 1 for root, 1 for left subtree, 1 for right)**

**After third rebalance:**




**(5 pts – 1 for root, 1 for 17, 1 for 56, 2 for rest)**

**5)** (9 points) **Binary Tree Traversals**




Give the preorder, inorder, and postorder traversals of the binary tree shown above.

**Preorder:**

**37, 88, 4, 15, 23, 49, 63, 45, 32, 17, 19, 6, 3, 26 (3 pts)**

**Inorder:**

**4, 15, 88, 49, 23, 45, 63, 32, 37, 19, 3, 6, 26, 17 (3 pts)**

**Postorder:**

**15, 4, 49, 45, 32, 63, 23, 88, 3, 26, 6, 19, 17, 37 (3 pts)**

**If two traversals are switched, take off 3 points total. If all three are switched, take off 6 points total.**
**If a majority of a traversal is correct, take off 1 point.**
**If some of a traversal is correct, but less than half, take off 2 points.**