You must do all 5 problems in this section of the exam.

Partial credit cannot be given unless all work is shown and is readable. Be complete, yet concise, and above all be neat. Do your rough work on the last page.
1) (11 points) **Recursion** Write a **recursive** function that sorts its input array values of length n as follows:

1) Finds the maximum item in the array in indexes 0 through n-1.
2) Swaps this element with the one stored in index n-1.
3) Recursively sorts the array values of length n-1.

```c
void rec_sort(int values[], int n)
{
    if (n < 2) return; // 2 pts - 0 also valid base case.
    int maxIndex = 0; // 1 pt
    int i;
    for (i=1; i<n; i++) // 1 pts
        if (values[i] > values[maxIndex]) // 1 pts
            maxIndex = i; // 1 pt
    int temp = values[n-1]; // 1 pt
    values[n-1] = values[maxIndex]; // 1 pt
    values[maxIndex] = temp; // 1 pt
    // 2 pts
    sort(values, n-1);
}
```
2) (10 points) **Summations**

a) Determine a closed-form solution for the following sum in terms of $n$: $\sum_{k=5}^{2n} (3k - 2)$.

\[
\sum_{k=5}^{2n} (3k - 2) = \sum_{k=1}^{2n} (3k - 2) - (1 + 4 + 7 + 10) = \frac{3(2n)(2n + 1)}{2} - 2(2n) - 22
\]

\[
= 3n(2n + 1) - 4n - 22 = 6n^2 + 3n - 4n - 22 = 6n^2 - n - 22
\]

**Grading:** 1 pt for small sum, 1 pt for large sum, 1 pt for final answer

b) Determine a closed-form solution for the following sum in terms of $n$: $\sum_{i=0}^{n} \left( 2 \sum_{j=n+1}^{3n} (i + j) \right)$

\[
\sum_{i=0}^{n} \left( 2 \sum_{j=n+1}^{3n} (i + j) \right) = \sum_{i=0}^{n} \left( 2 \left( \sum_{j=n+1}^{3n} i + \sum_{j=n+1}^{3n} j \right) \right)
\]

\[
= \sum_{i=0}^{n} \left( 2i(2n) + 2 \left( \sum_{j=1}^{3n} j - \sum_{j=1}^{n} j \right) \right)
\]

\[
= \sum_{i=0}^{n} \left( 4in + 2 \left( \frac{3n(3n + 1)}{2} - \frac{n(n + 1)}{2} \right) \right) (1 \text{ pt for each component, 3 pts total})
\]

\[
= \sum_{i=0}^{n} \left( 4in + 3n(3n + 1) - n(n + 1) \right)
\]

\[
= \sum_{i=0}^{n} \left( 4in + n[3(3n + 1) - (n + 1)] \right)
\]

\[
= \sum_{i=0}^{n} \left( 4in + n[9n + 3 - n - 1] \right)
\]

\[
= \sum_{i=0}^{n} \left( 4in + n[9n + 3 - n - 1] \right)
\]

\[
= \sum_{i=0}^{n} \left( 4in + n(8n + 2) \right)
\]

\[
= \frac{4n(n + 1)n}{2} + n(8n + 2)(n + 1) (1 \text{ pt for other sum, 2 pt for simplify})
\]

\[
= 2n^2(n + 1) + n(8n + 2)(n + 1)
\]

\[
= 2n(n + 1)[n + (4n + 1)]
\]

\[
= 2n(n + 1)(5n + 1) = 10n^3 + 12n^2 + 2n (1 \text{ pt final answer, either form})
\]
3) (10 points) **Stack Applications** Transform the following infix expression into its equivalent postfix expression using a stack. Show the contents of the stack at the indicated points 1, 2 and 3 in the infix expressions.

\[
( \frac{A}{B} + C ) - D * ( E - F / G )
\]

Grading: 2 pts for each stack, 4 pts for the whole expression (partial credit allowed.)

Resulting postfix expression:

\[
A \ B / C + D \ E \ F \ G / - * -
\]
4) (10 points) **AVL Trees** Draw the resulting AVL tree after inserting the following items (in this order) into an initially empty AVL tree: 56, 17, 13, 88, 27, 67, 35, 20, 5, 28. Show the tree after each step that requires a rebalance. (There are three of these steps.)

**After first rebalance:**
```
    17
   / \  
13  56
```

(2 pts – 1 for root, 1 for rest)

**After second rebalance:**
```
    56
   / \ 
 17  88
```

```
/ \ / 
13 27 67
```

(3 pts – 1 for root, 1 for left subtree, 1 for right)

**After third rebalance:**
```
    27
   / \ 
 17  56
```

```
/ \ / 
13 20 35 88
```

```
/ / / 
5 28 67
```

(5 pts – 1 for root, 1 for 17, 1 for 56, 2 for rest)
5) (9 points) **Binary Tree Traversals**

Give the preorder, inorder, and postorder traversals of the binary tree shown above.

**Preorder:**

37, 88, 4, 15, 23, 49, 63, 45, 32, 17, 19, 6, 3, 26 (3 pts)

**Inorder:**

4, 15, 88, 49, 23, 45, 63, 32, 37, 19, 3, 6, 26, 17 (3 pts)

**Postorder:**

15, 4, 49, 45, 32, 63, 23, 88, 3, 26, 6, 19, 17, 37 (3 pts)

If two traversals are switched, take off 3 points total. If all three are switched, take off 6 points total.  
If a majority of a traversal is correct, take off 1 point.  
If some of a traversal is correct, but less than half, take off 2 points.