Computer Science Foundation Exam

December 14, 2001

Section I B

No Calculators!

KEY

Name: _____

SSN: _____

In this section of the exam, there are three (3) problems

You must do all of them.

The weight of each problem in this section is indicated with the problem. The algorithms in this exam are written in a combination of pseudocode and programming language notation. Any algorithms that you are asked to produce should use a syntax that is clear and unambiguous. Partial credit cannot be given unless all work is shown.

As always, be complete, yet concise, and above all be neat. Credit cannot be given when your results are unreadable.

(4, 10%) You are given two global arrays **A** and **B**, each including a range of locations from **1** to **n**. Arrays **A** and **B** are already populated with arbitrary integer values, while array C has been initialized to zero at each location. Write a **recursive** subroutine called **prob4** that will compute the sum of **A**[I] and **B**[I] and store it in **C**[I], for each value of I ranging from 1 to **m**. Your subroutine should require a single parameter to accomplish its task.

Example: A = [1, 2, 4, 5] and B = [6, 3, 2, 4] the call **prob4(2)** would produce C = [7, 5, 0, 0]

Either form is acceptable

```
//calculates in order 1…m
procedure prob4( m: integer)
    if (m >= 1)
        { prob4(m-1);
          C[m] = A[m] + B[m];
        }
    endif
endprocedure


//calculuates in order m…1
procedure prob4 (m: integer)
   if (m >= 1)
       {C[m] = A[m] + B[m];
        prob4(m-1);
       }
   endif
endprocedure
```

(5, 18%) Find the closed form expression in terms of the parameter N (and M where indicated) for each of the following summations:

a)

$$\sum_{i=0}^{N}(4i-2) = 4\sum_{i=0}^{N} i - 2\sum_{i=0}^{N} 1 = 4\left[\frac{n(n+1)}{2}\right] - 2(n+1) = 2n^2 + 2n - 2n - 2 = 2n^2 - 2$$

Give the value of this expression for N = 38.

$$\sum_{i=0}^{38}(4i-2) = 2(38)^2 - 2 = 2886$$

b)

$$\sum_{i=1}^{3N+4}(6i+3) = 6\times\sum_{i=1}^{3N+4} i + 3\times\sum_{i=1}^{3N+4} 1 = 6\left[\frac{(3N+4)(3N+5)}{2}\right] + 3[3N+4]$$

$$= 3\times\left[9N^2 + 15N + 12N + 20\right] + 9N + 12$$

$$= 27N^2 + 45N + 36N + 60 + 9N + 12$$

$$= 27N^2 + 90N + 72$$

c)

$$\sum_{i=N}^{M}(5i-4) = \left(5\sum_{i=N}^{M} i - 5\sum_{i=1}^{N-1} i\right) - \left(4\sum_{i=1}^{M} 1 - 4\sum_{i=1}^{N-1} 1\right)$$

$$= \left(5\left[\frac{M(M+1)}{2}\right] - 5\left[\frac{(N-1)N}{2}\right]\right) - \left(4M - 4(N-1)\right)$$

$$= \tfrac{5}{2}(M^2 + M) - \tfrac{5}{2}(N^2 - N) - (4M - 4N + 4)$$

$$= \tfrac{5}{2}M^2 - \tfrac{3}{2}M - \tfrac{5}{2}N^2 + \tfrac{13}{2}N - 4$$

Give the value of this expression for N=40 and M = 68.

c)

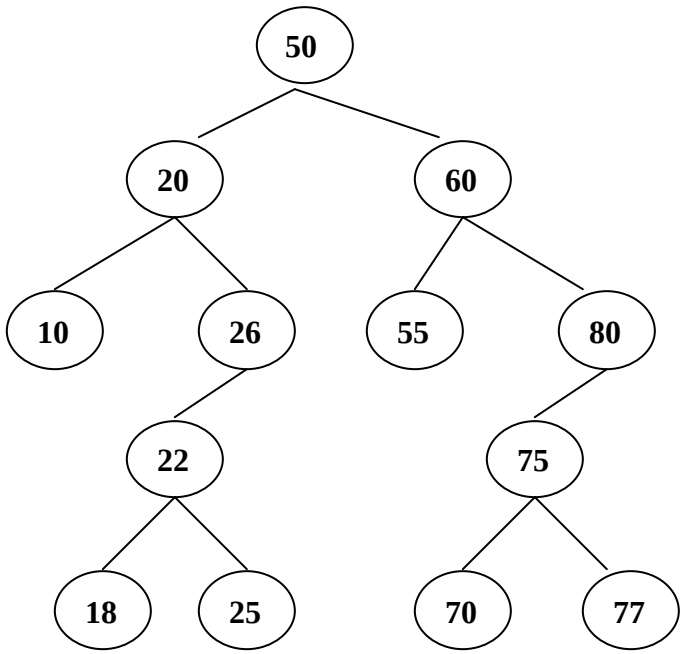
$$\sum_{i=40}^{65}(5i-4) = \left(5\sum_{i=40}^{65} i - 5\sum_{i=1}^{39} i\right) - \left(4\sum_{i=1}^{65} 1 - 4\sum_{i=1}^{39} 1\right)$$

$$= \left(5\left[\frac{65(66)}{2}\right] - 5\left[\frac{39(40)}{2}\right]\right) - \left((4(65)) - (4(39))\right)$$

$$= 5(2145) - 5(780) - 260 + 156$$

$$= 10725 - 3900 - 260 + 156 = 6721$$

(6, 18%) Given the following Binary Tree, answer the questions below :



a) Is this a valid Binary Search Tree? (circle one) YES NO

b) List the nodes of this tree in the order that they are visited in a preorder traversal:

50	20	10	26	22	18	25	60	55	80	75	70	77

first node visited — last node visited

c) Perform the following procedure on the tree above, listing the output in the spaces below and leaving any unused spaces blank. Assume that the procedure is initially called with: **P6(root, 20)** and that the tree nodes and pointers are defined as:

```
tree_node definesa record
    data isoftype Num
    left, right isoftype ptr toa tree_node
endrecord
tree_ptr isoftype ptr toa tree_node
```

```
procedure P6 (node_ptr isoftype in tree_ptr,  key isoftype in Num)
  if (node_ptr <> NULL) then
      P6(node_ptr^.left, (node_ptr^.data - key))
      P6(node_ptr^.right,  (node_ptr^.data + key))
      if (node_ptr^.data > key) then
            print(node_ptr^.data)
      endif
  endif
```

```
endprocedure
```

10 25 22 55 77 75 50