Computer Science Foundation Exam

August 18, 2000
Solution for CS 1 sections

(1, 20%) Given the following array of numbers and algorithm, answer the questions below. Assume that the global array \( X[1..n] \) is correctly declared and contains the values shown. Assume that the procedure was called with \( S(1, 6) \).

<table>
<thead>
<tr>
<th>Array X</th>
<th>4</th>
<th>5</th>
<th>2</th>
<th>6</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>position</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

```plaintext
procedure S(i, j : integer)
    a, b, y, z : integer
    a \leftarrow 0
    b \leftarrow 0
    y \leftarrow 0
    z \leftarrow 0
    while (i < j) do
        if (X[i] < X[j]) then
            z \leftarrow z + j
            X[i] \leftarrow X[i] + i
            i \leftarrow i + 1
            y \leftarrow y + X[i]
        else
            y \leftarrow y + i
            X[j] \leftarrow X[j] + j
            z \leftarrow z + X[j]
            j \leftarrow j - 1
        endif
    endwhile
    if (a <= b) then
        a \leftarrow X[i]
    else
        b \leftarrow X[j]
    endif
endprocedure
```

a) Show the array \( X \) after the procedure has completed execution?

<table>
<thead>
<tr>
<th>Array X</th>
<th>5</th>
<th>7</th>
<th>5</th>
<th>6</th>
<th>8</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>position</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

b) What value will the following variables contain after the \texttt{while} loop is finished?

\[
\begin{array}{ccccccc}
  a & b & y & 17 & z & 33 \\
\end{array}
\]
The following are Postfix expressions. All values are single decimal digits and the operations are addition "+", subtraction "-", multiplication "*" and division "/".
In each box below the Postfix expression, show ONLY the contents of the stack at the indicated point in the Postfix string (point A, B or C). Put the final answer in the blank. If the Postfix string is invalid, carry the operations as far as possible and write “invalid” as the answer.

a) 7 2 1 3 + A * - 5 3 + B * 2 / 6 C * = \(-24\)

\[
\begin{array}{|c|c|c|}
\hline
A & B & C \\
\hline
4 & 8 & -4 \\
2 & -1 & 6 \\
7 & & \\
\hline
\end{array}
\]

b) 3 5 - 8 2 A / 4 6 - 7 2 B - * C + * = \(12\)

\[
\begin{array}{|c|c|c|}
\hline
A & B & C \\
\hline
2 & -2 & 4 \\
7 & 4 & -10 \\
2 & -2 & -2 \\
8 & & \\
\hline
\end{array}
\]

Next to each Postfix expression, circle one answer to indicate if it is a valid Postfix string or not: (no extra credit for providing the answer, if it is valid)

\begin{align*}
\text{c)} \quad & 7 \ 5 \ 3 \ - \ + \ 3 \ / \ * \ 4 \ 2 \ + \quad \text{Invalid} \\
\text{d)} \quad & 4 \ 3 \ - \ 2 \ 3 \ * \ 5 \ 8 \ - \ + \ 4 \ - \ * \ 2 \ + \quad \text{Valid}
\end{align*}
(3, 20%) Answer each of the following "timing" questions concerning an algorithm of a particular order and a data set of a particular size. Assume that the run time is affected only by the size of the data set and not its composition.

a) For an $O(n^3)$ algorithm, one data set with $n = 5$ takes 250 seconds.

How long will it take for a data set with $n = 3$? 

$$\frac{5^3}{250} = \frac{3^3}{x} \Rightarrow x = \frac{27 \times 250}{125} = 54$$

54 seconds

b) For an $O(n \log_2 n)$ algorithm, one data set with $n = 8$ takes 96 seconds.

If you used a different-sized data set and it took 32 seconds, how large must that data set be?

$$\frac{8 \log(8)}{96} = \frac{n \log(n)}{32} \Rightarrow n \log(n) = 8; n = 4$$

n = 4

c) For an $O(2^n)$ algorithm, a friend tells you that it took 8 seconds to run on her data set. You run the same program, on the same machine, and your data set with $n = 7$ takes 64 seconds.

What size was her data set?

$$\frac{2^7}{8} = \frac{2^n}{64} \Rightarrow 2^n = 8 \times 128 / 64; n = \log(16) = 4$$

n = 4

Given the following pseudocode segment, answer the questions below for an arbitrary $n$:

```
x \leftarrow 0
for i \leftarrow 1 to (2*n) do
    for j \leftarrow 1 to (3*n) do
        x \leftarrow x + i
```

d) What is the Order of this pseudocode segment? $O(n^3)$

e) What will be the value of x when the for loops end? $6n^3 + 3n^2$

$$\sum_{i=1}^{2n} \sum_{j=1}^{3n} i = 3n \sum_{i=1}^{2n} i = 3n \frac{2n(2n+1)}{2} = 6n^3 + 3n^2$$
(4, 10%) In the space below, write a **recursive** algorithm called **PrintOdd**, that prints only the odd numbers from 1 to \( n \) in **increasing** order. The initial value of \( n \) may be either even or odd.

\[
\text{procedure } \text{PrintOdd}(n) \\
\quad \text{realPrint}(1, n); \\
\text{endprocedure;}
\]

\[
\text{procedure } \text{realPrint}(\text{current}, n) \\
\quad \text{if } \text{current} \leq n \text{ then} \\
\quad \quad \text{print current;} \\
\quad \quad \text{realPrint}(\text{current}+2, n); \\
\quad \text{endif;} \\
\text{endprocedure}
\]

There are, of course, many other correct solutions

------------------------------------------------------------------

(5, 18%) Find the closed form or exact value for the following:

( \( n \) is an arbitrary positive integer):

a) \[
\sum_{i=1}^{2n-1} (6i + 5) = 12n^2 + 4n - 5
\]

\[
= 6 \sum_{i=1}^{2n-1} i + 5 \cdot (2n - 1) = 6(2n)(2n-1)/2 + 10n - 5 = 12n^2 + 4n - 5
\]

b) \[
\sum_{i=0}^{60} (2ni - 4) = 3660n - 244
\]

\[
\sum_{i=0}^{60} (2ni - 4) = 2n \sum_{i=0}^{60} i - 4 \cdot 61 = 2n(61)(60)/2 - 244 = 3660n - 244
\]

c) \[
\sum_{i=40}^{100} (3i - 6) = 12444
\]

\[
\sum_{i=40}^{100} (3i - 6) = 3 \sum_{i=0}^{60} (i + 40 - 2) = 3(61)(60)/2 + 3(61)(38) = (3)(61)(30 + 38) = 12444
\]
Given the following Binary Tree, answer the questions below:

a) Is this a valid Binary Search Tree? (circle one)  **No**

b) List the nodes of this tree in the order that they are visited in a **postorder** traversal:

6  9  12  17  14  24  27  21  19

---

c) Perform the following procedure on the tree above, listing the output in the spaces below and leaving any unused spaces blank. Assume that the procedure is initially called with **Problem_6(root, 22)** and that the tree nodes and pointers are defined as:

```
tree_node defines record
  data isoftype Num
  left, right isoftype ptr toa tree_node
endrecord

procedure Problem_6(node_ptr isoftype in tree_ptr, key isoftype in Num)
  if (node_ptr <> NULL) then
    if (node_ptr^.data = key) then
      print(key)
    elseif (node_ptr^.data > key) then
      print(node_ptr^.data)
      Problem_6(node_ptr^.left, key)
    else
      print(node_ptr^.data)
      Problem_6(node_ptr^.right, key)
  endif
endprocedure
```

**Problem_6(19)  21  27  24**