

# Computer Science Foundation Exam

May 3, 2002

Section II A

## DISCRETE STRUCTURES

**NO books, notes, or calculators may be used,  
and you must work entirely on your own.**

**Name:** \_\_\_\_\_

**SSN:** \_\_\_\_\_

**In this section of the exam, there are two (2) problems.**

**You must do both of them.**

**Each counts for 25% of the total exam grade.**

**Show the steps of your work carefully.**

**Problems will be graded based on the completeness of  
the solution steps and not graded based on the answer alone**

**Credit cannot be given when your results are  
unreadable.**

## FOUNDATION EXAM (DISCRETE STRUCTURES)

Answer two problems of Part A and two problems of Part B. Be sure to show the steps of your work including the justification. The problem will be graded based on the completeness of the solution steps (including the justification) and **not** graded based on the answer alone. NO books, notes, or calculators may be used, and you must work entirely on your own.

### PART A: Work both of the following problems (1 and 2).

1) Prove by using induction on  $n$  that for all positive integers  $n > 0$ ,

$$\sum_{i=1}^n \frac{(i-1)2^i}{i(i+1)} = \frac{2^{n+1}}{n+1} - 2$$

2) For both parts of this question, let  $A$ ,  $B$  and  $C$  be finite sets. (Note that  $\neg$  denotes the complement of a set.)

a) Prove or disprove: if  $A \subseteq B$  and  $A \subseteq C$ , then  $\neg(B \cap C) \subseteq \neg A$ .

b) Prove or disprove: if  $A - B = B - C$ , then  $A = \emptyset$ .

Solution to Problem 1:

2) Prove by using induction on  $n$  that for all positive integers  $n > 0$ ,

$$\sum_{i=1}^n \frac{(i-1)2^i}{i(i+1)} = \frac{2^{n+1}}{n+1} - 2$$

Solution to Problem 2:

For both parts of this question, let  $A$ ,  $B$  and  $C$  be finite sets. (Note that  $\neg$  denotes the complement of a set.)

- a) Prove or disprove: if  $A \subseteq B$  and  $A \subseteq C$ , then  $\neg(B \cap C) \subseteq \neg A$ .
- b) Prove or disprove: if  $A - B = B - C$ , then  $A = \emptyset$ .

**Computer Science Foundation Exam**  
**May 3, 2002**

**Section II B**

**DISCRETE STRUCTURES**

**NO books, notes, or calculators may be used,  
and you must work entirely on your own.**

**Name:** \_\_\_\_\_

**SSN:** \_\_\_\_\_

**In this section of the exam, there are two (4) problems.**

**You must do two (2) of them.**

**Each counts for 25% of the total exam grade.**

**You must clearly identify the problems you are solving.**

**Show the steps of your work carefully.**

**Problems will be graded based on the completeness of  
the solution steps and not graded based on the answer alone**

**Credit cannot be given when your results are**

**unreadable.**

**PART B: Work any two of the following problems (3 through 6).**

**3)** Suppose  $R, S \subseteq A \times A$  are two symmetric relations on a set  $A$ . Prove or disprove each of the following propositions.

- a) the relation  $R \circ S$  is symmetric.
- b) the relation  $R \circ S \cup S \circ R$  is symmetric

**4)** Suppose  $f: A \rightarrow B$  and  $g: B \rightarrow A$  are two functions. Assume that for any element  $y \in B$ ,  $(f \circ g)(y) = y$ , but there is at least one element  $x \in A$  such that  $(g \circ f)(x) \neq x$ . Prove that  $f$  is onto, but not one to one.

**5)** Consider the following relation on a set of positive integers  $n > 0$ .

$$R = \{(a, b) \mid \gcd(a, b) = 15 \text{ and } \text{lcm}(a, b) = 180\}.$$

Explicitly write out all pairs in  $R$  and show that no other pairs are members of  $R$ .

**6)** Consider six-digit numbers with all distinct digits that do NOT start with 0. Answer the following questions about these numbers. Leave the answer in factorial form.

- a) How many such numbers are there?
- b) How many of these numbers contain a 3 but not 6?
- c) How many of these numbers contain either 3 or 6 (or both)?

Solution to Problem \_\_\_\_ (Please write in the problem number 3,4,5 or 6 you are solving)

Solution to Problem \_\_\_\_ (Please write in the problem number 3,4,5 or 6 you are solving)