In this section of the exam, there are three (3) problems

You must do all of them.

The weight of each problem in this section is indicated with the problem. The algorithms in this exam are written in a combination of pseudocode and programming language notation. Any algorithms that you are asked to produce should use a syntax that is clear and unambiguous. Partial credit cannot be given unless all work is shown.

As always, be complete, yet concise, and above all be neat. Credit cannot be given when your results are unreadable.
Write a recursive procedure, called prob4, that will correctly sum only the odd integers which occur, within the first m locations of an array X. You may assume that X is a global array which includes locations that range from 1 to n and is already populated with integers. Assume that m ≤ n. You may also assume that the sum s, is initialized to 0. Assume the initial call is prob4(m, s). You may use pseudocode, C, Java or Pascal syntax but points will be deducted if your meaning is not clear. You may also assume the existence of a function odd which takes an integer parameter and returns true if the parameter is odd and false otherwise.

One possible solution is:

```plaintext
procedure prob4(int m, int s)
{
    if m > 0
    {
        if odd(X[m])
            s = s + X[m];
        prob4(m-1, s);
    } //endif
} //end procedure prob4
```
(5, 18%) Find the closed form expression in terms of the parameter $N$ (and $M$ where indicated) for each of the following summations:

\[ a) \sum_{i=0}^{N} (5i + 6) = 5 \sum_{i=0}^{N} i + 6 \sum_{i=0}^{N} 1 = 5 \frac{n(n + 1)}{2} + 6(n + 1) = \frac{5n^2 + 5n + 12n + 12}{2} = \frac{5n^2 + 17n + 12}{2} \]

Give the value of this expression for $N = 27$.

When $N = 27$ we have: \( \frac{5(27)^2 + 17(27) + 12}{2} = \frac{4116}{2} = 2058 \)

\[ b) \sum_{i=1}^{2N-5} (4i + 5) = 4 \sum_{i=1}^{2N-5} i + 5 \sum_{i=1}^{2N-5} 1 = 4 \left[ \frac{(2n - 5)(2n - 4)}{2} \right] + 5(2n - 5) = 8n^2 - 26n + 15 \]

\[ c) \sum_{i=N}^{M} (3i - 6) = 3 \sum_{i=n}^{m} i - 6 \sum_{i=n}^{m} 1 = 3 \left[ \sum_{i=1}^{m} i - \sum_{i=1}^{n-1} i \right] - 6 \left[ \sum_{i=1}^{m} 1 - \sum_{i=1}^{n-1} 1 \right] = 3 \left[ \frac{m(m + 1)}{2} \right] - 3 \left[ \frac{(n - 1)(n)}{2} \right] - 6(m) + 6(n - 1) = \frac{3m^2 - 9m - 3n^2 + 15n - 12}{2} \]

Give the value of this expression for $N=10$ and $M = 20$.

\[ c) \sum_{i=10}^{20} (3i - 6) = \frac{3(20)^2 - 9(20) - 3(10)^2 + 15(10) - 12}{2} = \frac{858}{2} = 429 \]
(6, 18%) Given the following Binary Tree, answer the questions below:

![Binary Tree Diagram]

a) Is this a valid Binary Search Tree? (circle one)  YES      NO

b) List the nodes of this tree in the order that they are visited in a postorder traversal:

<table>
<thead>
<tr>
<th>first node visited</th>
<th>last node visited</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>60</td>
</tr>
<tr>
<td>15</td>
<td></td>
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<tr>
<td>10</td>
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</tr>
<tr>
<td>70</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td></td>
</tr>
</tbody>
</table>


c) Perform the following procedure on the tree above, listing the output in the spaces below and leaving any unused spaces blank. Assume that the procedure is initially called with: P6(root, 30) and that the tree nodes and pointers are defined as:

```
tree_node defines a record
data isoftype Num
  left, right isoftype ptr to a tree_node
endrecord
tree_ptr isoftype ptr to a tree_node
```

```plaintext
procedure P6 (node_ptr isoftype in tree_ptr, key isoftype in Num)
  if (node_ptr <> NULL) then
    P6(node_ptr^.left, (node_ptr^.data - key))
    if (node_ptr^.data > key) then
      print(node_ptr^.data)
    endif
    P6(node_ptr^.right, (node_ptr^.data + key))
  endif
endif
endprocedure
```

| 10 | 12 | 15 | 60 | 69 |   |   |   |   |   |