Problem #1 [12 points total]

Must write a recursive function. Many students will attempt this problem and use no recursion at all. Give only 4 points credit if their solution is correct but not recursive.

1. [12 points] The digital root of a number is obtained by summing up the digits repeatedly until only a single digit remains. For example, the digital root of 7854 is obtained by computing 7+8+5+4 which is 24. Next the digits 2 and 4 are summed to yield 6. Since it is a single digit, this forms the digital root of the given number. Write a recursive function root(int n) that returns the digital root of the argument without making use of any loop constructs (like for, while etc.) in your function. Partial credit may be given in case your function includes such constructs.

root(n)
if ( n/10 ) == 0 )
    return ( n % 10 );
else
    return root( n % 10 + root ( n /10 ) );

Grading Policy:

Give 12 points if recursive solution is correct

Give maximum 6 points if iterative solution is correct

Give partial credit, if there are minor mistakes, but the student has got the basic idea about how to approach the solution. (Deduct 2 or 3 from max pts. Based on solution type.)
Problem #2 [12 points total]

These types of problems require the students to either determine the closed form of a summation expression and/or find an exact value of a summation if summation range is known. Since calculators are not allowed to be used during the exam, we do not require the students to reduce to exact numbers. They are allowed to leave the answer in terms of expressions, provided, of course, that the expression will evaluate to the correct result. Using part (a) as an example the solution:

\[
\sum_{i=5}^{N} (7i - 3) = \left( 7 \sum_{i=1}^{N} i - 7 \sum_{i=1}^{N} i \right) - \left( 3 \sum_{i=1}^{N} 1 - 3 \sum_{i=1}^{N} 1 \right) = \frac{7n^2 + 7n}{2} - \frac{7(4)(5)}{2} - 3N + 12
\]

is sufficient for full credit. Each problem is worth 5 points and 4 of these points are to be assigned to properly setting up the problem, i.e., showing the technique by which the closed form is generated. Only one point of each problem is assigned to the arithmetic portion. Common mistakes are improper closed forms for the summations shown below as well as missing minus signs which cause subsequent subtraction operations to become additions.

- math errors – deduct 1 point if simple addition error and deduct two points if involve a sign error or improper algebra.
- improper setup or technique for producing the closed form – deduct 3 to 6 points depending upon the severity of the offence.
- wrong closed form used – deduct 2 or 3 points for each offence.

\[
\sum_{i=1}^{N} i = \frac{N(N + 1)}{2} \quad \sum_{i=0}^{N} i = \frac{N(N + 1)}{2} \quad \sum_{i=1}^{N} 1 = N \quad \sum_{i=0}^{N} 1 = N + 1
\]

Problem #3 [12 points total]

Part (a) [6 points] – If they set up the problem correctly, but don’t do the math correctly, deduct only 1 point. Deduct 1 to 2 points for simple math errors.

Part (b) [6 points] – Most common error will be either using wrong exponent value or improperly determining the value of a power of 2. Deduct 1 point for these types of errors if the problem is otherwise set-up correctly.
Problem #4 [14 points total]

Deduct 3 points for first incorrect value, 2 points for second incorrect value and 1 point for each additional incorrect value.