

Computer Science Foundation Exam

March 9, 2001

Section II A

DISCRETE STRUCTURES

**NO books, notes, or calculators may be used,
and you must work entirely on your own.**

Name: _____

SSN: _____

In this section of the exam, there are two (2) problems.

You must do both of them.

Each counts for 25% of the total exam grade.

Show the steps of your work carefully.

**Problems will be graded based on the completeness of
the solution steps and not graded based on the answer alone**

Credit cannot be given when your results are unreadable.

FOUNDATION EXAM (DISCRETE STRUCTURES)

Answer two problems of Part A and two problems of Part B. Be sure to show the steps of your work including the justification. The problem will be graded based on the completeness of the solution steps (including the justification) and **not** graded based on the answer alone. NO books, notes, or calculators may be used, and you must work entirely on your own.

PART A: Work both of the following problems (1 and 2).

- 1) Let A, B , and C be sets such that $C \subset B$ (i.e., C is a proper subset of B , or possibly $C = B$). Use appropriate set theoretic laws and theorems to prove that $(A - B) \cup (B - C) = \overline{C} \cap (A \cup B)$. Be sure to explain each step of your proof.
- 2) Prove by induction that for $n \geq 1$,

$$1(1!) + 2(2!) + \dots + n(n!) = (n + 1)! - 1,$$

where the notation $!$ means factorial, i.e., $1! = 1$, and for $k \geq 2$, $k! = k(k - 1)!$.

Solution to Problem 1:

Let A , B , and C be sets such that $C \subset B$. Use appropriate laws and theorems to prove that

$$(A - B) \cup (B - C) = \overline{C} \cap (A \cup B).$$

Solution to Problem 2:

Prove by induction that for $n \geq 1$

$$1(1!) + 2(2!) + \dots + n(n!) = (n + 1)! - 1.$$

Computer Science Foundation Exam

March 9, 2001

Section II B

DISCRETE STRUCTURES

**NO books, notes, or calculators may be used,
and you must work entirely on your own.**

Name: _____

SSN: _____

In this section of the exam, there are two (4) problems.

You must do two (2) of them.

Each counts for 25% of the total exam grade.

You must clearly identify the problems you are solving.

Show the steps of your work carefully.

**Problems will be graded based on the completeness of
the solution steps and not graded based on the answer alone**

Credit cannot be given when your results are unreadable.

PART B: Work any two of the following problems (3 through 6).

3) An ice cream shop lets its customers create their orders. Each customer can choose up to four scoops of ice cream from 10 different flavors. In addition, they can add any combination of the 7 toppings to their ice cream. (Note: Please leave your answer in factorials, combinations, and powers.)

a) If a customer is limited to at most two scoops of the same flavor, how many possible orders with exactly 4 scoops and up to 5 toppings can the customer make? (Assume each order has at least one topping.)

b) Suzanne wants to make 7 separate orders for ice cream. Each order will have exactly 1 scoop and 1 topping. If no flavor or topping is requested more than once, how many combinations of orders can Suzanne make?

4) Let R be a relation with $R \subset A \times A$, where $|A| = 5$. Answer the following questions, giving detailed justifications for your answers.

a) Give an example of a non-empty relation that is both symmetric and anti-symmetric. (Let the set $A = \{a, b, c, d, e\}$.)

b) How many relations R are symmetric?

5) Let $f: A \rightarrow B$ be a function, and $g: B \rightarrow C$ be a function.

a) If $g \circ f: A \rightarrow C$ is surjective (onto), prove that g is also surjective (onto).

b) Give a small example of functions f and g such that f is an injection (one-to-one) and g is a surjection (onto), but $g \circ f$ is not an injection (one-to-one).

6) Let $D(n)$ be the n digit (decimal) number consisting of just 1s. Use induction (or any other method) to show that $11 \mid D(2n)$, for all integers $n > 0$. (Note that we can recursively define $D(n)$ as follows: $D(n) = 10D(n-1) + 1$ for $n > 1$, $D(1) = 1$. Explicitly, $D(n) = (10^n - 1) / 9$. Also, note that $a \mid b$ means that a divides b evenly, i.e., $b \% a = 0$.)

Solution to Problem ____ (Please write in the problem number 3,4,5 or 6 you are solving)

Solution to Problem ____ (Please write in the problem number 3,4,5 or 6 you are solving)