Access Game: Fair Bandwidth Sharing in Distributed Systems through a Game Theoretic Framework

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Overview

- Fairness and QoS
- Distributed MAC protocols
- Selfish Users
- Equilibrium
- Complete and incomplete information game

Quality of Service (QoS)

- Network resource requirement for the users

  -bandwidth, buffer space etc.

- Different types of users have different types of requirements
- Bandwidth is the primary resource

  -this is where MAC protocols come into play

  -bandwidth can be obtained *only* through accessing the medium

Fairness in Bandwidth Allocation

- Idea: users should get resources proportional to their weightage [1]
- Why:together with proper call admission control algorithms, can ensure satisfactory QoS
- Easy to implement in centralized systems
- Not so in distributed systems

    -several challenges

    -contention is the main problem

Medium Access Control (MAC)

- Classifications:

  1. Centralized

     -scheduling, decision taken by an authority

     -example: HiperLAN

  2. Distributed

     - contention-based

     - examples: ALOHA, CSMA

     - we concentrate on CSMA protocols

Carrier Sense Multiple Access (CSMA)

- Sense the channel
- Classifications: based on the actions taken when the channel is sensed busy
  1. Non-persistent CSMA
  2. 1-persistent CSMA
  3. *p*-persistent CSMA
- We use *p*-persistent CSMA

p-persistent CSMA

- Listen to the channel

  -if the channel is busy, keep on listening

  -if the channel is idle, transmission is possible and users enter the contention period

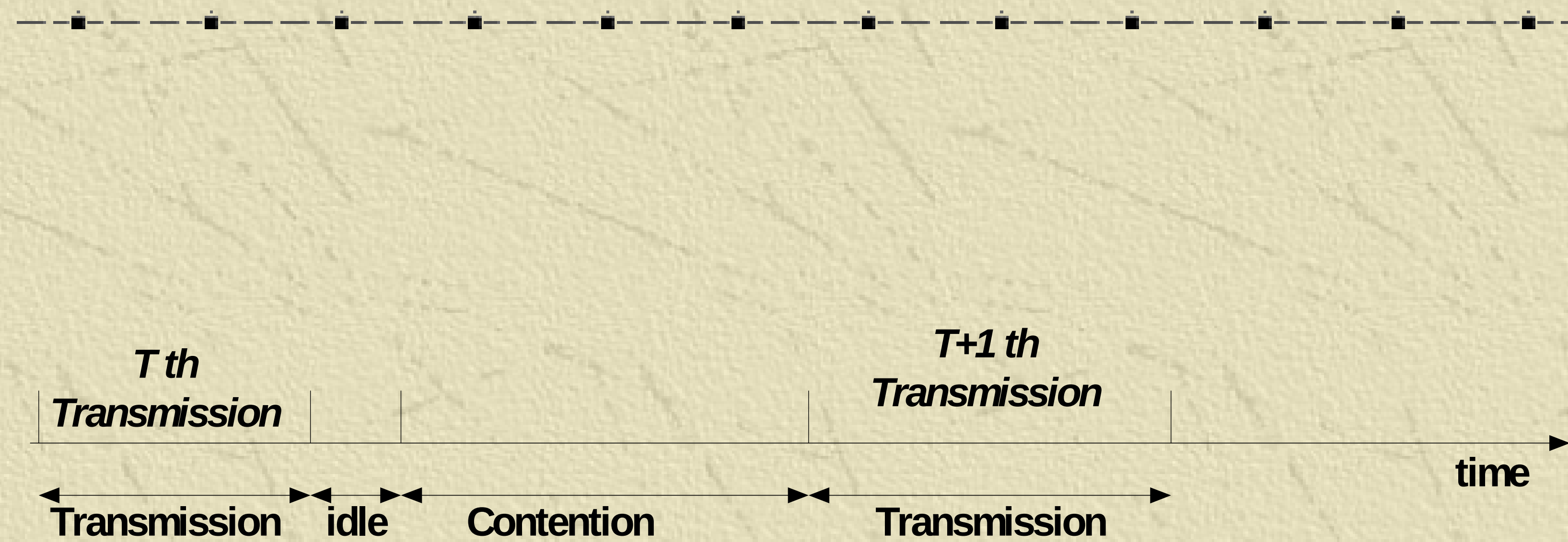
- Contention

  -transmit with probability p

  -with probability ($1-p$), do not transmit and stop listening for a small period of time

  -contention is eventually resolved (details omitted)

p-persistent CSMA (contd.)



p-persistent CSMA (contd.)

- At the beginning of contention, a user has two options
  - -*transmit* with probability p
  - -*wait* with probability $(1-p)$
- If more than one user transmits, there is collision

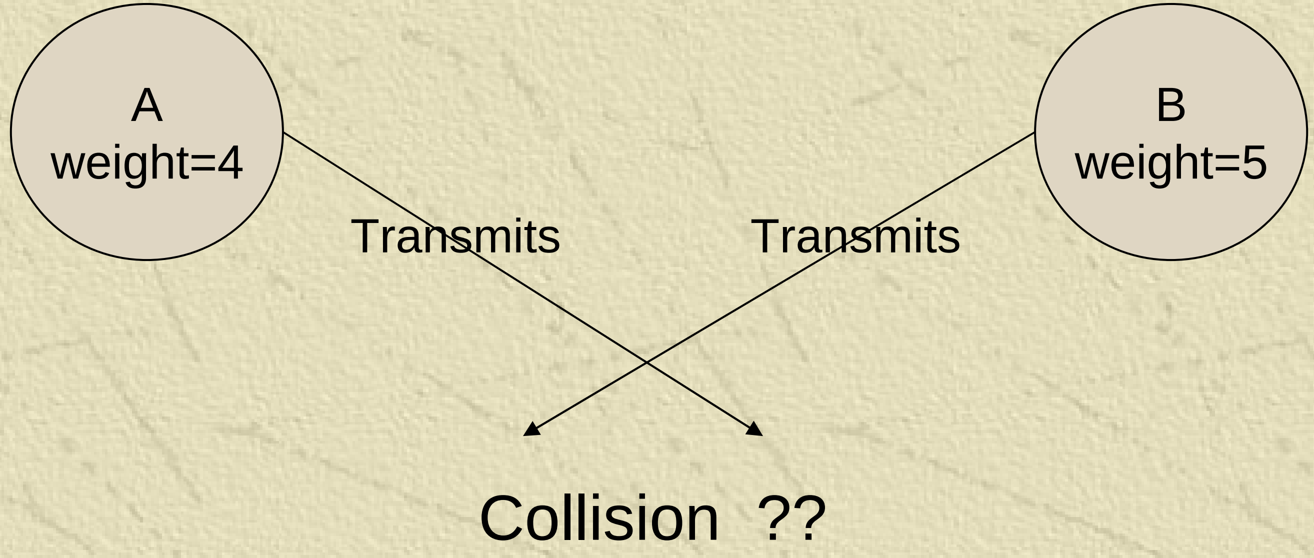
Distributed MAC &Fairness

- Distributed MAC protocols are contention-based
- No user is guaranteed to access the medium
- However, priorities of different users can be taken into account
- Let us look at an example

Two Users in a Distributed Scenario

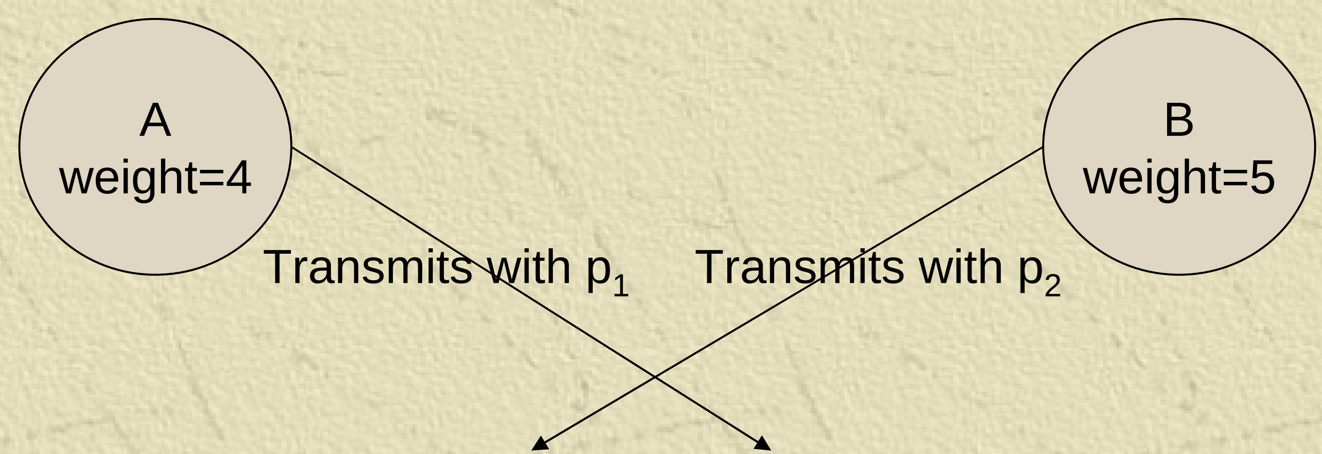
Packet Arrival

Both Transmit



Collision

- Contention-based medium access:
  - -always a possibility of collision
  - -depends on the *probability* with which users transmit
- If $p_1=p_2=1$, there is always collision
  - -this was the case with early ALOHA
- $p_1,p_2<1$ for *p*-persistent CSMA



Probability(collision)= $p_1 p_2$

Success Probability

- Each of the user has a non-zero probability of successfully capturing the medium

$$P_A = p_1(1 - p_2)$$

$$P_B = p_2(1 - p_1)$$

- For fairness, the success probability should be proportional to the weightage

Fairness in Distributed Access

- Fairness can be defined as

$$\frac{P_A}{w_A} = \frac{P_B}{w_B}$$

- Need to compute the transmission probabilities that result in fairness
- These can be computed easily in this simple example

Fair transmission probabilities

- For fairness

$$p_1 = \frac{\sqrt{w_1}}{\sqrt{w_1} + \sqrt{w_2}} = \frac{\sqrt{4}}{\sqrt{4} + \sqrt{5}}$$

$$p_2 = \frac{\sqrt{w_2}}{\sqrt{w_1} + \sqrt{w_2}} = \frac{\sqrt{5}}{\sqrt{4} + \sqrt{5}}$$

- Check

$$\frac{\frac{\sqrt{w_1}}{\sqrt{w_1} + \sqrt{w_2}}[1 - \frac{\sqrt{w_1}}{\sqrt{w_1} + \sqrt{w_2}}]}{w_1} = \frac{1}{[\sqrt{w_1} + \sqrt{w_2}]^2} = \frac{\frac{\sqrt{w_2}}{\sqrt{w_1} + \sqrt{w_2}}[1 - \frac{\sqrt{w_2}}{\sqrt{w_1} + \sqrt{w_2}}]}{w_2}$$

Questions

- Existence: is there a set of transmission probabilities that would result in fairness for a general case?
- Uniqueness: if there is a solution, is it unique?
- Computation: how can these probabilities be computed in a distributed fashion?

Present State

- CSMA/CD and CSMA/CA
- CSMA/CA can take into consideration the relative weightages of different users.
- Extensive experimental studies have shown that these protocols do not result in fairness.

Independence and Selfishness

- Distributed Framework:

  -users should be given independence to choose their individual transmission strategy

  -this introduces new challenges: selfish users

- Selfish Users:

  -If users are given independence to choose their transmission probability, they might misuse it

  -they might act greedily i.e. try to get as much as bandwidth as possible

Summing Up

- Fairness requires independence for users to choose transmission policy
- However, independent users would try to maximize their own chance of success
- Natural setting for Game-Theory: access game

Access Game

- Each selfish user wants to maximize its own chance of success
- We look into two solution concepts:

  -Nash Equilibrium [2]; no constraints

  -Constrained Nash Equilibrium [3]; fairness conditions will be the constraints,

Strategy Space and Outcome

- Actions

  -transmit and wait

  -outcome of the game depends on the collective actions taken by all the users

- Outcomes

  -if only one user transmits, that user “succeeds”

  -if more than one user transmits, there is collision and “failure:

  -if no user transmits, that transmission period is “wasted”

Payoff and Utility

- Payoff:

  -if a user succeeds, it receives a payoff of “1”

  -”0” otherwise

- Utility Function:

  -expected payoff

$$u_i = p_i \prod_{j \neq i}^{n} (1 - p_j) = \frac{p_i}{(1 - p_i)} g$$

  -concave with respect to p_i

Nash Equilibrium

- Nash Equilibrium for the access game is

$$p_i = 1 \qquad \forall i$$

  -pure strategy equilibrium

- Inefficient Solution:

  -probability of success is nil

  -rational users would be willing to adhere to a set constraints if it benefits them

Constrained Nash Equilibrium

- The fairness conditions are the constraints

$$\frac{p_1 \prod_{j \neq 1}^{n} (1 - p_j)}{w_1} = .. \frac{p_i \prod_{j \neq i}^{n} (1 - p_j)}{w_i} = .. \frac{p_n \prod_{j = 1}^{n-1} (1 - p_j)}{w_n}$$

- Therefore, if the constrained Nash Equilibrium exists, it will satisfy fairness
- As $p_i = 0, 1$ satisfies the constraints trivially, we do not consider them

Solution

- Solution for the game exists for concave utility functions: u_i is concave wrt p_i

  -not *strictly* concave but concave

- Each user solves for

$$\frac{\partial u_i}{\partial p_i} = 0$$

and

$$\frac{p_i / 1 - p_i}{w_i} = \frac{p_j / 1 - p_j}{w_j} \qquad \forall i, j$$

Solution

- Eventually,

$$\sum_{i=1}^{n} p_i = 1$$

$$\frac{p_1/(1-p_1)}{w_1} = .. \frac{p_i/(1-p_i)}{w_i} = .. \frac{p_n/(1-p_n)}{w_n} = \frac{1}{K}$$

$$p_i \neq 0,1 \qquad \forall i$$

- Therefore, $\sum_{i=1}^{n} \frac{w_i}{K+w_i} = 1$

Unique Solution

$$\sum_{i=1}^{n} \frac{w_i}{K + w_i} = 1$$

- The above equation has a unique solution in *K(>0)*
- Therefore, transmission probabilities are also unique
- It can be shown that the throughput is also optimized.

Discussion

- Nash Equilibrium is inefficient
- Constrained Nash Equilibrium is the suitable concept
- A unique solution exists for *any* case.
- The solution is Pareto-optimal

    -user/user:fairness

    -user/system:throughput

Incomplete Information

- We have considered complete information games
- Incomplete information games are more realistic.
- Approximation: information gathering and dissemination.

Approximation

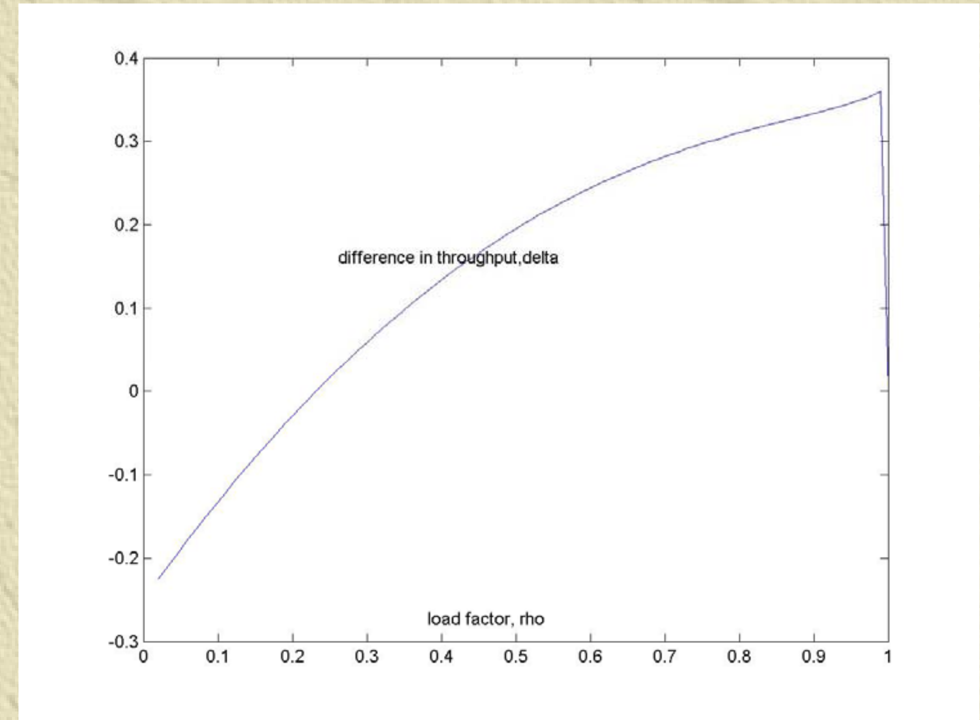
- Typically there is a registration authority
- Information gathering:

  -when a user joins or leaves the system, the registration authority updates notes that event

  -increase the number of users of different types accordingly
- Information dissemination:

  -number of users of different classes is broadcast at the beginning of each contention period.

Throughput Comparison



Difference in throughput as a function of load

Issues

- Node-based fairness as opposed to flow-based fairness
- Stability of the protocol
- Back-off window-based “fair” transmission strategy

References

1. Demers, A., Keshav,S.,and Shenker,S.; *"Analysis and Simulation of a fair-queueing algorithm"*, SIGCOMM 1989.
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3. Rosen, J. "*Existence and uniqueness of equilibrium point for concave n-person games*", Econometric:33,pp.520-534, 1965