LAGRANGE INTERPOLATING POLYNOMIALS ①

(reformulation of Newton's polynomials that avoids computation of divided differences (and associated math errors!))

$$f_n(x) = \sum_{i=0}^{n} \underline{L_i(x)}\, f(x_i)$$

where $L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \left(\dfrac{x - x_j}{x_i - x_j} \right)$

	x	$f(x)$
0	–	–
1	–	–
2	–	–
3	–	–

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \left(\frac{x - x_j}{x_i - x_j} \right) = \boxed{\text{patterns below are used}}$$

n is order of polynomial to be (

Say $i = 0$

$$L_0(x) = \frac{\cancel{(x - x_0)}\,(x - x_1)(x - x_2)(x - x_3)\cdots(x - x_n)}{\cancel{(x_0 - x_0)}\,(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)\cdots(x_0 - x_n)}$$

$i = 1$

$$L_1(x) = \frac{(x - x_0)}{(x_1 - x_0)} \frac{\cancel{(x - x_1)}}{\cancel{(x_1 - x_1)}} \frac{(x - x_2)}{(x_1 - x_2)} \frac{(x - x_3)}{(x_1 - x_3)} \cdots \frac{(x - x_n)}{(x_1 - x_n)}$$

$i = 2$

$$L_2(x) = \frac{(x - x_0)}{(x_2 - x_0)} \frac{(x - x_1)}{(x_2 - x_1)} \frac{\cancel{(x - x_2)}}{\cancel{(x_2 - x_2)}} \frac{(x - x_3)}{(x_2 - x_3)} \cdots \frac{(x - x_n)}{(x_2 - x_n)}$$

look at formats $\quad$ $n=1$

first order $\quad f_1(x) = L_0(x) \cdot f(x_0) + L_1(x) \cdot f(x_1)$

(linear)

$$= \left(\frac{x - x_1}{x_0 - x_1}\right) \cdot f(x_0) + \left(\frac{x - x_0}{x_1 - x_0}\right) \cdot f(x_1)$$

$n=2$

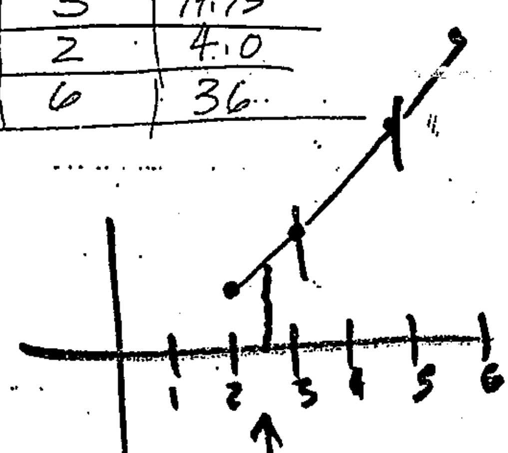
2ND order (quadratic) $\quad f_2(x) = L_0(x) \cdot f(x_0) + L_1(x) \cdot f(x_1) + L_2(x) \cdot f(x_2)$

$$=$$

GO TO NEXT PAGE FOR $n=2$ and $n=3$

NOTE - "BRACKET" POINT ORDER IS CRITICAL; SAY WE WANTED $f(x=2.5)$. THEN WE WOULD NEED TO RE ORDER INPUT POINT ORDER

X NAME	~~x~~	f(x)
X_0	3	5.25
X_1	5	19.75
X_2	2	4.0
X_3	6	36



First order
linear

$$f_1(x) = \frac{x - x_1}{x_0 - x_1} f(x_0) + \frac{x - x_0}{x_1 - x_0} f(x_1)$$

2nd order
quadratic

$$\boxed{\begin{aligned} f_2(x) = {} & \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) \\ & + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) \\ & + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2) \end{aligned}}$$

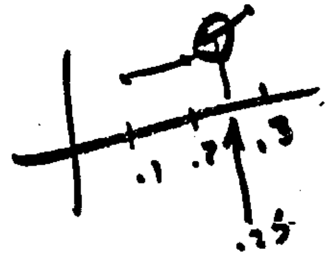
cubic
3rd order

$$\begin{aligned} f_3(x) = {} & \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0) \\ & + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} f(x_1) \\ & + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} f(x_2) \\ & + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} f(x_3) \end{aligned}$$

④

EXAMPLE
WIND TUNNEL

i	x	$f(x)$
0	.1	.04
1	.2	.09
2	.3	.13



use linear to find
$f(x = .25)$

so, redefine points

i	x	$f(x)$
0	.2	.09
1	.3	.13

$$f_1(x) = \frac{x - x_1}{x_0 - x_1} f(x_0) + \frac{x - x_0}{x_1 - x_0} f(x_1)$$

$$f_1(x) = \frac{(x - .3)}{.2 - .3}(.09) + \frac{(x - .2)}{(.3 - .2)}(.13)$$

$$f_1(x) = (x - .3)\left(\frac{.09}{-.1}\right) + (x - .2)\left(\frac{.13}{.1}\right)$$

$$f_1(x) = (x - .3)(-.9) + (x - .2)(1.3)$$

$-.9x + .27$
$+1.3x - .26$

$$f_1(x) = .4x + .01$$

$f_1(.25) = (.4)(.25) + .01$
$= .10 + .01 = .11$

EXAMPLE PROBLEM w/ LAGRANGE POLYS

USE EARLIER WIND TUNNEL

i	x	$f(x)$
0	.1	.04
1	.2	.09
2	.3	.13

3 data points, fit a quadratic $n=2$

$$f_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \cdot f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} \cdot f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} \cdot f(x_2)$$

$$f_2(x) = \frac{(x-.2)(x-.3)}{(.1-.2)(.1-.3)}(.04) + \frac{(x-.1)(x-.3)}{(.2-.1)(.2-.3)}(.09) + \frac{(x-.1)(x-.2)}{(.3-.1)(.3-.2)}(.13)$$

$$= x^2 - .5x + .06 \,\frac{(.04)}{(.02)} \quad \text{etc} + \quad +$$

CALCULATE f(3.5) USING THE LAGRANG… POLYNOMIALS OF ORDERS 1 THROUGH 3.

16

$\boxed{12.8}$

$x_0 = 3 \quad f(x_0) = 5.25$

$x_1 = 5 \quad f(x_1) = 19.75$

$x_2 = 2 \quad f(x_2) = 4$

$x_3 = 6 \quad f(x_3) = 36$

(Eq. 12.22)

$$f_1(3.5) = \frac{(3.5-5)}{(3-5)} 5.25 + \frac{(3.5-3)}{(5-3)} 19.75$$

$$= 8.875$$

(Eq. 12.23)

$$f_2(3.5) = \frac{(3.5-5)(3.5-2)}{(3-5)(3-2)} 5.25 + \frac{(3.5-3)(3.5-2)}{(5-3)(5-2)} 19.75 + \frac{(3.5-3)(3.5-5)}{(2-3)(2-5)} 4$$

$$= 7.375$$

(Eq. $\frac{12.20}{12.21}$ with n = 3)

$$f_3(3.5) = \frac{(3.5-5)(3.5-2)(3.5-6)}{(3-5)(3-2)(3-6)} 5.25 + \frac{(3.5-3)(3.5-2)(3.5-6)}{(5-3)(5-2)(5-6)} 19.75 + \frac{(3.5-3)(3.5-5)(3.5-6)}{(2-3)(2-5)(2-6)} 4 + \frac{(3.5-3)(3.5-5)(3.5-2)}{(6-3)(6-5)(6-2)} 36$$

$$= 7.09375$$

6