Su 97
EGN 3420 Quiz Name ____________________

SHOW ALL WORK!!

Open Notes - No calculators (leave results of all calculations in terms of fractions)

Given the system of equations $A\underline{x}=\underline{b}$ below,

$$
\begin{array}{ccccccc}
x & + & y & + & z & = & 9 \\
2x & - & y & + & 3z & = & 14 \\
x & + & 3y & - & 2z & = & 0
\end{array}
$$

1. Show there is a unique solution without solving for it. (5 pts)
2. Find the solution by Gauss Elimination. (5 pts)
3. Find the solution using the Gauss-Jordan Method. (5 pts)
4. Find the solution using $\underline{x}=A^{-1}\underline{b}$ where the inverse is obtained in either of the two ways discussed in class. (5 pts)

1. $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 1 & 3 & -2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & -3 & 1 \\ 0 & 2 & -3 \end{vmatrix} = -3\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & -1/3 \\ 0 & 2 & -3 \end{vmatrix} = -3\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & -1/3 \\ 0 & 0 & -7/3 \end{vmatrix}$

$= -3(-7/3)\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & -1/3 \\ 0 & 0 & 1 \end{vmatrix} = 7$ => unique solution

2. $(A \vdots \underline{b}) = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 2 & -1 & 3 & 14 \\ 1 & 3 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -3 & 1 & -4 \\ 0 & 2 & -3 & -9 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 1 & -1/3 & 4/3 \\ 0 & 2 & -3 & -9 \end{array}\right)$

$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 1 & -1/3 & 4/3 \\ 0 & 0 & -7/3 & -35/3 \end{array}\right) \sim \begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 1 & -1/3 & 4/3 \\ 0 & 0 & 1 & 5 \end{array}\right) \end{array}$ => $z = 5$

$y - \frac{1}{3}z = \frac{4}{3}$

$y = \frac{4}{3} + \frac{1}{3}(5) = \frac{9}{3} = 3$

soln. $\underline{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}$

$x + y + z = 9$

$x = 9 - 3 - 5 = 9 - 8 = 1$

3. $(A \,\vdots\, \underline{b}) = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 2 & -1 & 3 & 14 \\ 1 & 3 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 1 & -1/3 & 4/3 \\ 0 & 0 & 1 & 5 \end{array}\right)$

$\sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right) \sim \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right) \end{array} \Rightarrow \begin{array}{l} x = 1 \\ y = 3 \\ z = 5 \end{array}$

4.

$$\begin{array}{c} \begin{array}{cc} A & I \end{array} \\ \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & -1 & 3 & 0 & 1 & 0 \\ 1 & 3 & -2 & 0 & 0 & 1 \end{array}\right] \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -3 & 1 & -2 & 1 & 0 \\ 0 & 2 & -3 & -1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1/3 & 2/3 & -1/3 & 0 \\ 0 & 2 & -3 & -1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1/3 & 2/3 & -1/3 & 0 \\ 0 & 0 & -7/3 & -7/3 & 2/3 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1/3 & 2/3 & -1/3 & 0 \\ 0 & 0 & 1 & 1 & -2/7 & -3/7 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 2/7 & 3/7 \\ 0 & 1 & 0 & 1 & -3/7 & -1/7 \\ 0 & 0 & 1 & 1 & -2/7 & -3/7 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 5/7 & 4/7 \\ 0 & 1 & 0 & 1 & -3/7 & -1/7 \\ 0 & 0 & 1 & 1 & -2/7 & -3/7 \end{array}\right]$$

$$\Rightarrow A^{-1} = \frac{1}{7}\begin{bmatrix} -7 & 5 & 4 \\ 7 & -3 & -1 \\ 7 & -2 & -3 \end{bmatrix}$$

$$\underline{x} = A^{-1}\underline{b}$$

$$= \frac{1}{7}\begin{bmatrix} -7 & 5 & 4 \\ 7 & -3 & -1 \\ 7 & -2 & -3 \end{bmatrix}\begin{bmatrix} 9 \\ 14 \\ 0 \end{bmatrix}$$

$$= \frac{1}{7}\begin{bmatrix} 7 \\ 21 \\ 35 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$