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EGN 3420 Exam 2 Name__________________

SHOW ALL WORK!

Problem 1 (20 pts)

Solve the following system of equations using the Gauss Jordan Elimination Method.

$$\begin{array}{rcrcrcrcr} a & + & b & + & c & + & d & = & 0 \\ 2a & - & b & - & c & + & d & = & 3 \\ a & + & 3b & + & 2c & - & 4d & = & -1 \\ 5a & + & 2b & - & 2c & + & d & = & 7 \end{array}$$

Work Area

$$(A|\underline{b}) = \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 2 & -1 & -1 & 1 & 3 \\ 1 & 3 & 2 & -4 & -1 \\ 5 & 2 & -2 & 1 & 7 \end{array}\right] \approx \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & -3 & -3 & -1 & 3 \\ 0 & 2 & 1 & -5 & -1 \\ 0 & -3 & -7 & -4 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1/3 & -1 \\ 0 & 2 & 1 & -5 & -1 \\ 0 & -3 & -7 & -4 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1/3 & -1 \\ 0 & 0 & -1 & -17/3 & 1 \\ 0 & 0 & -4 & -3 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1/3 & -1 \\ 0 & 0 & 1 & 17/3 & -1 \\ 0 & 0 & -4 & -3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1/3 & -1 \\ 0 & 0 & 1 & 17/3 & -1 \\ 0 & 0 & 0 & 59/3 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1/3 & -1 \\ 0 & 0 & 1 & 17/3 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

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Problem 2 (25 pts)

Consider the following system of equations:

$$\begin{array}{rrrrrcr}
u & + v & + w & & & = & 2 \\
2u & - v & & - 3x & - 3y & = & 1 \\
u & - 2v & + 3w & - 3x & - 3y & = & -1 \\
u & & + w & - x & - y & = & 1 \\
3u & + v & + 5w & - 2x & - 2y & = & 4
\end{array}$$

A) Show the equations are consistent by transforming the augmented matrix (A|b) into its Echelon Form by performing a sequence of elementary row operations.

B) There are 2 arbitrary unknowns. Explain why.

C) Without solving for a solution determine if x and y are both arbitrary.

D) Repeat Part C) for x and w.

E) What conclusion can be drawn from the results of Part C) and D)?

A)

$$\left[\begin{array}{rrrrr|r}
1 & 1 & 1 & 0 & 0 & 2 \\
2 & -1 & 0 & -3 & -3 & 1 \\
1 & -2 & 3 & -3 & -3 & -1 \\
1 & 0 & 1 & -1 & -1 & 1 \\
3 & 1 & 5 & -2 & -2 & 4
\end{array}\right] \sim \begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 2 \\
0 & -3 & -2 & -3 & -3 & -3 \\
0 & -3 & 2 & -3 & -3 & -3 \\
0 & -1 & 0 & -1 & -1 & -1 \\
0 & -2 & 2 & -2 & -2 & -2
\end{bmatrix}$$

Augmented Matrix

$$\sim \begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 2 \\
0 & 1 & 0 & 1 & 1 & 1 \\
0 & -3 & 2 & -3 & -3 & -3 \\
0 & -3 & -2 & -3 & -3 & -3 \\
0 & -2 & 2 & -2 & -2 & -2
\end{bmatrix} \sim \begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 2 \\
0 & 1 & 0 & 1 & 1 & 1 \\
0 & 0 & 2 & 0 & 0 & 0 \\
0 & 0 & -2 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0
\end{bmatrix} \sim \begin{array}{cccccc}
u & v & w & x & y & \\
\end{array}
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 2 \\
0 & 1 & 0 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

B) Number of arbitrary unknowns $= 5 - 3 = 2$

C)
$$\begin{array}{ccc} u & v & w \end{array}$$
$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$
x & Y arbitrary

D)
$$\begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{vmatrix} = 0$$
x & w not arbitrary

E) w is not arbitrary

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Problem 3 (25 pts)

Find the value(s) of K in the system of equations below which result in an infinite number of solutions. Use only methods discussed in class.

$$
\begin{aligned}
x_1 + 4x_2 + 7x_3 &= -5 \\
x_1 + x_2 + x_3 &= 1 \\
2x_1 + 3x_2 + 4x_3 &= 0 \\
x_1 - x_2 + Kx_3 &= 0
\end{aligned}
$$

$$
\left[\begin{array}{ccc|c} 1 & 4 & 7 & -5 \\ 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 0 \\ 1 & -1 & K & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 4 & 7 & -5 \\ 0 & -3 & -6 & 6 \\ 0 & -5 & -10 & 10 \\ 0 & -5 & K-7 & 5 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 4 & 7 & -5 \\ 0 & 1 & 2 & -2 \\ 0 & 1 & 2 & -2 \\ 0 & -5 & K-7 & 5 \end{array}\right]
$$

$$
\sim \left[\begin{array}{ccc|c} 1 & 4 & 7 & -5 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & K+3 & -5 \end{array}\right]
$$

There ~~will~~ cannot be an infinite number of solutions. If $K+3=0$

$K=-3$

$\Rightarrow$ Equations inconsistent

Check

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Problem 4 (30 pts)

Consider the system of equations $A\underline{x}=b$ below.

1. Find the inverse of A and the solution $\underline{x}=A^{-1}\underline{b}$.
2. Perform two iterations of the Gauss-Seidel method and compute the true errors in x, y and z, i.e. $(E_T)_x$, $(E_T)_y$ and $(E_T)_z$ at the end of the second iteration. Start with an initial guess of $x_0 = y_0 = z_0 = 1$.

$$\begin{aligned} 5x + y + z &= 45 \\ x + 5y + z &= 25 \\ -x + y + 5z &= -55 \end{aligned}$$

1.
$$\left[\begin{array}{ccc|ccc} 5 & 1 & 1 & 1 & 0 & 0 \\ 1 & 5 & 1 & 0 & 1 & 0 \\ -1 & 1 & 5 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 5 & 1 & 0 & 1 & 0 \\ 5 & 1 & 1 & 1 & 0 & 0 \\ -1 & 1 & 5 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 5 & 1 & 0 & 1 & 0 \\ 0 & -24 & -4 & 1 & -5 & 0 \\ 0 & 6 & 6 & 0 & 1 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 5 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1/6 & 1/6 \\ 0 & 0 & 20 & 1 & -1 & 4 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 5 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1/6 & 1/6 \\ 0 & 0 & 1 & 1/20 & -1/20 & 1/5 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 5 & 0 & -1/20 & 21/20 & -1/5 \\ 0 & 1 & 0 & -1/20 & 13/60 & -1/30 \\ 0 & 0 & 1 & 1/20 & -1/20 & 1/5 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/5 & \overset{-1/30}{\cancel{-11/5}} & -1/30 \\ 0 & 1 & 0 & -1/20 & 13/60 & -1/30 \\ 0 & 0 & 1 & 1/20 & -1/20 & 1/5 \end{array}\right]$$

$$\underline{x} = A^{-1}\underline{b} = \frac{1}{60}\begin{bmatrix} 12 & -2 & -2 \\ -3 & 13 & -2 \\ 3 & -3 & 12 \end{bmatrix}\begin{bmatrix} 45 \\ 25 \\ -55 \end{bmatrix} = \frac{1}{60}\begin{bmatrix} 600 \\ 300 \\ -600 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ -10 \end{bmatrix}$$

(over)

2. $x = 9 - \frac{1}{5}y - \frac{1}{5}z$

$y = 5 - \frac{1}{5}x - \frac{1}{5}z$

$z = -11 + \frac{1}{5}x - \frac{1}{5}y$

$x_0 = y_0 = z_0 = 1$

$x_1 = 9 - \frac{1}{5}y_0 - \frac{1}{5}z_0 = 9 - \frac{2}{5} = \frac{43}{5}$ (8.6)

$y_1 = 5 - \frac{1}{5}x_1 - \frac{1}{5}z_0 = 5 - \frac{1}{5}\left(\frac{43}{5}\right) - \frac{1}{5}(1) = \frac{77}{25}$ (3.08)

$z_1 = -11 + \frac{1}{5}x_1 - \frac{1}{5}y_1 = -11 + \frac{1}{5}\left(\frac{43}{5}\right) - \frac{1}{5}\left(\frac{77}{25}\right) = \frac{-1237}{125}$ (-9.896)

$x_2 = 9 - \frac{1}{5}y_1 - \frac{1}{5}z_1 = 9 - \frac{1}{5}\left(\frac{77}{25}\right) - \frac{1}{5}\left(\frac{-1237}{125}\right) = \frac{6477}{625}$ (10.3632)

$y_2 = 5 - \frac{1}{5}x_2 - \frac{1}{5}z_1 = 5 - \frac{1}{5}\left(\frac{6477}{625}\right) - \frac{1}{5}\left(\frac{-1237}{125}\right) = \frac{15333}{3125}$ (4.90656)

$z_2 = -11 + \frac{1}{5}x_2 - \frac{1}{5}y_2 = -11 + \frac{1}{5}\left(\frac{6477}{625}\right) - \frac{1}{5}\left(\frac{15333}{3125}\right) = \frac{-154823}{15625}$

(-9.908672)

$(E_T)_x = 10 - (10.3632)$

$= -0.3632$

$(E_T)_y = 5 - 4.90656$

$= 0.09344$

$(E_T)_z = -10 - (-9.908672)$

$= -0.091328$