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EGN 3420 Exam 1 Name ____________________

SHOW ALL WORK!

Problem 1 (25 pts)

Consider the function $f(x) = xe^x$. At $x=1$, the difference between the 2^{nd} order Taylor Series expansion of f(x) about some pt x_0 and the 1^{st} order Taylor Series expansion of f(x) about the same pt x_0 is equal to ¼.

1. Find an equation that can be used to solve for x_0.
2. Perform 2 iterations of the simple one point iteration method to determine x_0.

1. $f(x) = xe^x$

$f'(x) = xe^x + e^x = (x+1)e^x$

$f''(x) = (x+1)e^x + e^x = (x+2)e^x$

$$f_2(x) - f_1(x) = \frac{f''(x_0)(x-x_0)^2}{2!} = \frac{(x_0+2)e^{x_0}(x-x_0)^2}{2!}$$

$$\Rightarrow f_2(1) - f_1(1) = \frac{(x_0+2)e^{x_0}(1-x_0)^2}{2} = \frac{1}{4}$$

Ans. $2(x_0+2)e^{x_0}(1-x_0)^2 = 1$

2. $$x_0 = g(x_0) = \frac{e^{-x_0}}{2(1-x_0)^2} - 2$$

Initial guess $x_{0,0} = 0$

$$\Rightarrow x_{0,1} = g(x_{0,0}) = \frac{e^{-x_{0,0}}}{2(1-x_{0,0})^2} - 2 = \frac{e^0}{2(1-0)^2} - 2 = \frac{1}{2} - 2 = -\frac{3}{2}$$

$$x_{0,2} = g(x_{0,1}) = \frac{e^{-x_{0,1}}}{2(1-x_{0,1})^2} - 2 = \frac{e^{-(-3/2)}}{2(1- -3/2)^2} - 2 = \frac{e^{3/2}}{2(5/2)^2} - 2$$

$$= -1.6415$$

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Problem 2 (25 pts) Display all answers to 3 places after the decimal point!

a) Apply the Bisection method to find the root of the function $f(x) = x - \frac{16}{x^3}$ located between $x_l=1$ and $x_u=4$. Compute the approximate and true relative errors expressed as a percent at the completion of the 3rd iteration.

b) Repeat Part a) using the Newton-Raphson method starting with an initial guess of $x_0=1$.

a) $f(x) = x - \frac{16}{x^3}$, $f(x_l) = f(1) = 1 - \frac{16}{1^3} = -15$

$f(x_u) = f(4) = 4 - \frac{16}{4^3} = 4 - \frac{16}{64} = 3.75$

iteration #1: $x_R = \frac{x_l + x_u}{2} = \frac{1+4}{2} = 2.5$

$f(x_R) = f(2.5) = 2.5 - \frac{16}{2.5^3} = 1.476$

$f(x_L) f(x_R) = (-)(+) < 0 \Rightarrow x_U \leftarrow x_R \Rightarrow x_U = 2.5$

iteration #2: $x_R = \frac{x_L + x_U}{2} = \frac{1+2.5}{2} = 1.75$

$f(x_R) = f(1.75) = 1.75 - \frac{16}{1.75^3} = -1.2354$

$f(x_L) f(x_R) = (-)(-) > 0 \Rightarrow x_L \leftarrow x_R \Rightarrow x_L = 1.75$

iteration #3: $x_R = \frac{x_L + x_U}{2} = \frac{1.75 + 2.5}{2} = 2.125$

$e_A = \frac{x_R^{new} - x_R^{old}}{x_R^{new}} \times 100 = \frac{2.125 - 1.75}{2.125} \times 100 = 17.647\%$

$e_T = \frac{R - x_R}{R} \times 100 = \frac{2 - 2.125}{2} \times 100 = -6.25\%$

Problem 2

b. $f(x) = x - \frac{16}{x^3}$, $f'(x) = 1 + \frac{48}{x^4}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$, $f(x_0) = x_0 - \frac{16}{x_0^3} = 1 - \frac{16}{1^3} = -15$

$f'(x_0) = 1 + \frac{48}{x_0^4} = 1 + \frac{48}{1^4} = 49$

$\Rightarrow x_1 = 1 - \frac{-15}{49} = \frac{64}{49} = 1.306122449$

$f(x_1) = \frac{64}{49} - \frac{16}{(64/49)^3} = -5.874602649$

$f'(x_1) = 1 + \frac{48}{(64/49)^4} = 17.49322796$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.306122449 - \frac{-5.874602649}{17.49322796} = 1.641943983$

$f(x_2) = 1.641943983 - \frac{16}{(1.641943983)^3} = -1.972528918$

$f'(x_2) = 1 + \frac{48}{(1.641943983)^4} = 7.604012571$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.641943983 - \frac{-1.972528918}{7.604012571} = 1.901350303$

$e_A = \frac{x_3 - x_2}{x_3} \times 100 = \frac{1.901350303 - 1.641943983}{1.901350303} \times 100 = 13.643\ \%$

$e_T = \frac{R - x_3}{R} \times 100 = \frac{2 - 1.901350303}{2} \times 100 = 4.932\ \%$

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Problem 3 (25 pts) Display all answers to 3 places after the decimal point!

a. Find the equation of the Lagrange polynomial that passes thru the data points (0,-2), (1,0), (2,2) and (4,6) taken from an unknown function f(x).

b. Use the polynomial for interpolation at x=3.

c. Compute the true error under the assumption that the true function f(x) is linear.

a. $f_3(x) = L_0(x)f(x_0) + L_1(x)f(x_1) + L_2(x)f(x_2) + L_3(x)f(x_3)$

$$= \left[\frac{(x-1)(x-2)(x-4)}{(0-1)(0-2)(0-4)}\right](-2) + 0 + \left[\frac{(x-0)(x-1)(x-4)}{(2-0)(2-1)(2-4)}\right](2)$$

$$+ \left[\frac{(x-0)(x-1)(x-2)}{(4-0)(4-1)(4-2)}\right](6)$$

b. $$f_3(3) = \left[\frac{(2)(1)(-1)}{(-1)(-2)(-4)}\right](-2) + \left[\frac{(3)(2)(-1)}{(2)(1)(-2)}\right](2) + \left[\frac{(3)(2)(1)}{(4)(3)(2)}\right](6)$$

$$= -\frac{1}{2} + 3 + \frac{3}{2}$$

$$= 4$$

c. The linear function thru all 4 data pts is $f(x) = 2x - 2$

$\Rightarrow f(3) = 2(3) - 2$

$= 4$

$E_T = f(3) - f_3(3)$

$= 4 - 4$

$= 0$

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Problem 4 (25 pts) Display all answers to 3 places after the decimal point!

1. Fit a second order Newton's interpolating polynomial $f_2(x)$ thru the data points $(-1, e^{-1})$, $(0,1)$ and $(1,e)$ from an unknown function $f(x)$.
2. Evaluate the resulting polynomial at $x=0.5$
3. Estimate the error in $f_2(0.5)$ by using an additional point at $(2,e^2)$.
4. Can we determine the true error at $x=0.5$? Explain.

1.

x	$f(x)$	Δ	Δ^2	Δ^3
-1	e^{-1}	$1-e^{-1}$	$\frac{e-2+e^{-1}}{2}$	$\left(\frac{e^2-3e+3-e^{-1}}{2}\right)/3$
0	1	$e-1$	$\frac{e^2-2e+1}{2}$	
1	e	e^2-e		
2	e^2			

$$f_2(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1)$$

$$= e^{-1} + (1-e^{-1})(x+1) + \left(\frac{e-2+e^{-1}}{2}\right)(x+1)x$$

2. $$f_2(0.5) = e^{-1} + (1-e^{-1})(1.5) + \left(\frac{e-2+e^{-1}}{2}\right)(1.5)(0.5)$$

$$= \frac{3e+6-e^{-1}}{8} = 1.723$$

3. Error in $f_2(0.5) = b_3(x-x_0)(x-x_1)(x-x_2)$

$b_3 = \frac{e^2-3e+3-e^{-1}}{6}$

$x = 0.5$

$$R_2 = \left(\frac{e^2-3e+3-e^{-1}}{6}\right)(0.5+1)(0.5)(0.5-1)$$

$$R_2 = f_3(0.5) - f_2(0.5)$$

$=$

4. No, because we don't know the true function $f(x)$.