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SHOW ALL WORK!

Problem 1 (25 pts)

For the system of equations

$$
\begin{array}{rcrcrcrcrcl}
 & & & & x_3 & + & x_4 & + & x_5 & = & 2 \\
3x_1 & + & 3x_2 & - & 2x_3 & + & x_4 & + & x_5 & = & 2 \\
5x_1 & + & 3x_2 & - & 2x_3 & + & 3x_4 & + & x_5 & = & 4 \\
x_1 & + & 2x_2 & + & x_3 & + & 2x_4 & + & 3x_5 & = & 5
\end{array}
$$

A) Find the Echelon matrix and determine if the equations are consistent.
B) If the equations are consistent, how many arbitrary unknowns are there?
C) Can x_1 and x_2 both be arbitrary? If they can, use the Gauss Jordan method applied to the system of equations with x_1 and x_2 on the right hand side to find x_3, x_4, and x_5 in terms of x_1 and x_2.

Work Area

A)

$$
\left(\begin{array}{ccccc|c} 0 & 0 & 1 & 1 & 1 & 2 \\ 3 & 3 & -2 & 1 & 1 & 2 \\ 5 & 3 & -2 & 3 & 1 & 4 \\ 1 & 2 & 1 & 2 & 3 & 5 \end{array}\right) \sim \left(\begin{array}{ccccc|c} 1 & 2 & 1 & 2 & 3 & 5 \\ 3 & 3 & -2 & 1 & 1 & 2 \\ 5 & 3 & -2 & 3 & 1 & 4 \\ 0 & 0 & 1 & 1 & 1 & 2 \end{array}\right)
$$

$$
\sim \left(\begin{array}{ccccc|c} 1 & 2 & 1 & 2 & 3 & 5 \\ 0 & -3 & -5 & -5 & -8 & -13 \\ 0 & -7 & -7 & -7 & -14 & -21 \\ 0 & 0 & 1 & 1 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{ccccc|c} 1 & 2 & 1 & 2 & 3 & 5 \\ 0 & 1 & 1 & 1 & 2 & 3 \\ 0 & -3 & -5 & -5 & -8 & -13 \\ 0 & 0 & 1 & 1 & 1 & 2 \end{array}\right)
$$

$$
\sim \left(\begin{array}{ccccc|c} 1 & 2 & 1 & 2 & 3 & 5 \\ 0 & 1 & 1 & 1 & 2 & 3 \\ 0 & 0 & -2 & -2 & -2 & -4 \\ 0 & 0 & 1 & 1 & 1 & 2 \end{array}\right) \sim
\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \end{array}
\left(\begin{array}{ccccc|c} 1 & 2 & 1 & 2 & 3 & 5 \\ 0 & 1 & 1 & 1 & 2 & 3 \\ 0 & 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)
$$

Echelon Form

Equations are consistent.

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B) $\left.\begin{matrix} n = 5 \\ m = 3 \end{matrix}\right\} \Rightarrow$ 2 arbitrary unknowns

c) x_1 & x_2 are arbitrary if $\begin{array}{c} \begin{matrix} x_3 & x_4 & x_5 \end{matrix} \\ \begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{vmatrix} \end{array} \neq 0$

Simplifying and evaluating the determinant,

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ 0 & -1 & -2 \end{vmatrix} = 2 - 1 = 1$$

$\therefore$ x_1 & x_2 <u>are</u> arbitrary

$$\begin{array}{c} \begin{matrix} x_3 & x_4 & x_5 & \end{matrix} \\ \left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 - x_1 - 2x_2 \\ 1 & 1 & 2 & 3 \quad - x_2 \\ 1 & 1 & 1 & 2 \end{array}\right) \end{array} \sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 - x_1 - 2x_2 \\ 0 & -1 & -1 & -2 + x_1 + x_2 \\ 0 & -1 & -2 & -3 + x_1 + 2x_2 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 - x_1 - 2x_2 \\ 0 & 1 & 1 & 2 - x_1 - x_2 \\ 0 & 0 & -1 & -1 \quad + x_2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 0 & 2 - x_1 + x_2 \\ 0 & 1 & 0 & 1 - x_1 \\ 0 & 0 & 1 & 1 - x_2 \end{array}\right)$$

$$\sim \begin{array}{c} \begin{matrix} x_3 & x_4 & x_5 & \end{matrix} \\ \left(\begin{array}{ccc|c} 1 & 0 & 0 & x_1 + x_2 \\ 0 & 1 & 0 & 1 - x_1 \\ 0 & 0 & 1 & 1 - x_2 \end{array}\right) \end{array}$$

SOLUTION:

$x_3 = x_1 + x_2$

$x_4 = 1 - x_1$

$x_5 = 1 - x_2$

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SHOW ALL WORK!

Problem 2 (25 pts)

Find the value(s) of K for which the system of equations below does not possess a unique solution.

$$
\begin{array}{ccccccccccc}
w & + & x & + & y & + & z & = & 6 \\
w & + & x & + & y & - & z & = & 0 \\
2w & - & x & + & y & & & = & 1 \\
w & - & 2x & + & Ky & + & 3z & = & 7
\end{array}
$$

Work Area

$A\underline{x} = \underline{b}$ where $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 2 & -1 & 1 & 0 \\ 1 & -2 & K & 3 \end{pmatrix}$

$A\underline{x} = \underline{b}$ does not have a unique solution if $|A| = 0$

$$|A| = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 2 & -1 & 1 & 0 \\ 1 & -2 & K & 3 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & -2 \\ 0 & -3 & -1 & -2 \\ 0 & -3 & K-1 & 2 \end{vmatrix} = \begin{vmatrix} 0 & 0 & -2 \\ -3 & -1 & -2 \\ -3 & K-1 & 2 \end{vmatrix}$$

$$0 = (-2)(-1)^{1+3} \begin{vmatrix} -3 & -1 \\ -3 & K-1 \end{vmatrix}$$

$$0 = -2\{(-3)(K-1) - 3\}$$

$$0 = -2(-3K)$$

$$0 = K$$

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Problem 3 (25 pts)

Solve the system of equations below using the Gauss-Seidel method. Start with an initial guess of $x^\circ = 1$, $y^\circ = 1$, $z^\circ = 1$. Stop when the magnitude of the true error in z falls below 0.1, i.e. $|3 - z^i| < 0.1$ since z=3 is the solution for z.

$$\begin{aligned} 4x - 2y + z &= 3 \\ x + 3y - z &= 4 \\ x - y + 4z &= 11 \end{aligned}$$

Keep all intermediate calculations in terms of fractions.

Work Area

$x = \frac{1}{4}(3 + 2y - z) \Rightarrow x^{i+1} = \frac{1}{4}(3 + 2y^i - z^i)$

$y = \frac{1}{3}(4 - x + z) \Rightarrow y^{i+1} = \frac{1}{3}(4 - x^{i+1} + z^i)$

$z = \frac{1}{4}(11 - x + y) \quad z^{i+1} = \frac{1}{4}(11 - x^{i+1} + y^{i+1})$

$x' = \frac{1}{4}(3 + 2y^\circ - z^\circ) = \frac{1}{4}(3 + 2 - 1) = 1$

$y' = \frac{1}{3}(4 - x' + z^\circ) = \frac{1}{3}(4 - 1 + 1) = \frac{4}{3}$

$z' = \frac{1}{4}(11 - x' + y') = \frac{1}{4}(11 - 1 + \frac{4}{3}) = \frac{17}{6}, \ |3 - z'| = |3 - \frac{17}{6}| = 0.166$

$x^2 = \frac{1}{4}(3 + 2y' - z') = \frac{1}{4}(3 + \frac{8}{3} - \frac{17}{6}) = \frac{17}{24}$

$y^2 = \frac{1}{3}(4 - x^2 + z') = \frac{1}{3}(4 - \frac{17}{24} + \frac{17}{6}) = \frac{147}{72}$

$z^2 = \frac{1}{4}(11 - x^2 + y^2) = \frac{1}{4}(11 - \frac{17}{24} + \frac{147}{72}) = \frac{111}{36}, \ |3 - z^2| = |3 - \frac{111}{36}| = 0.083$

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Problem 4 (25 pts)

Using Simpson's 1/3 Rule to approximate $\int_0^1 [1/(1+x)]\, dx$. The interval (0,1) is to be divided into 10 equal subintervals producing a step size of h = 0.1

....

Work Area

i	x_i	$f_i = f(x_i)$
0	0.0	1.0000
1	0.1	0.9091
2	0.2	0.8333
3	0.3	0.7692
4	0.4	0.7143
5	0.5	0.6667
6	0.6	0.6250
7	0.7	0.5882
8	0.8	0.5556
9	0.9	0.5263
10	1.0	0.5000

$$I = \int_0^1 \frac{dx}{1+x} \approx \frac{h}{3}\left\{f_0 + 4(f_1+f_3+f_5+f_7+f_9) + 2(f_2+f_4+f_6+f_8) + f_{10}\right\}$$

$$\approx \frac{0.1}{3}\left\{1 + 4(3.4595) + 2(2.7282) + 0.5\right\}$$

$$\approx 0.6931$$