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EGN 3420 EXAM 1 Name_______________

Problem 1 (40 pts)

Consider the function $f(x) = x^3 - 4x^2 + 6x - 4$. Fill in the tables below and stop when the true relative error falls below 15 % or after the fifth iteration, whichever comes first. Retain the maximum number of digits in all intermediate calculations; however round all displayed results in the tables to 3 places after the decimal point.

Show work for one iteration to receive partial credit!

A. Bisection Method

True Root, $R = 2$

$$x_R = \frac{x_L + x_U}{2} = \frac{0+10}{2} = 5.$$

Iteration #	x_L	x_U	x_R	$\lvert e_T \rvert$, %
1	0.000	10.000	5.000	150.000
2	0.000	5.000	2.500	25.000
3	0.000	2.500	1.250	37.500
4	1.250	2.500	1.875	6.250
5				

$$|e_T| = \left|\frac{R - x_R}{R}\right| \times 100 = \left|\frac{2-5}{2}\right| \times 100 = 150\%$$

B. False Position Method

$i = 0$:

$f(x_L) = f(0) = -4$

$f(x_U) = f(3) = 5$

Iteration #	x_L	x_U	x_R	$\lvert e_T \rvert$, %
1	0.000	3.000	1.333	33.333
2	1.333	3.000	1.548	22.581
3	1.548	3.000	1.701	14.950
4				
5				

$$x_R = x_L - f(x_L)\left[\frac{x_U - x_L}{f(x_U) - f(x_L)}\right] = 0 - (-4)\left(\frac{3-0}{5-(-4)}\right) = 1.33333333$$

$i = 1$:

$f(x_L) = f(1.333) = -0.740740741$

$$x_R = 1.33333333 - (-0.740740741)\left(\frac{3 - 1.33333333}{5-(-0.7407407)}\right) = 1.548387097$$

(over)

$i = 2$:

$$x_R = 1.548387097 - (-0.587425732)\left(\frac{3 - 1.548387097}{5 - (-0.587425732)}\right)$$

$$f(x_L) = f(1.548)$$

$$= 1.701000270$$

$$= -0.587425732$$

C. Simple One Point Iteration

$x_{i+1} = g(x_i)$ where $g(x) =$ $(4+4x^2-x^3)/6$

i	x_i	$\lvert e_T \rvert$, %
0	1.000	50.000
1	1.167	41.667
2	1.309	34.529
3	1.436	28.223
4	1.547	22.627
5	1.645	17.726

$x_1 = g(x_0)$

$= \dfrac{4+4(1)^2-1^3}{6} = \dfrac{7}{6}$

D. Newton Raphson Method

$f'(x) =$ $3x^2 - 8x + 6$

i	x_i	$\lvert e_T \rvert$, %
0	1.000	50.000
1	2.000	0.000
2		
3		
4		
5		

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} = x_i - \frac{x_i^3 - 4x_i^2 + 6x_i - 4}{3x_i^2 - 8x_i + 6}$$

$$i=0, \quad x_1 = 1 - \left(\frac{1^3 - 4(1)^2 + 6(1) - 4}{3(1)^2 - 8(1) + 6} \right) = 1 - \left(\frac{-1}{1} \right) = 2$$

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Problem 2 (20 pts)

Consider the table of values obtained from an unknown function f(x).

i	x_i	$f(x_i)$	Δ	Δ^2	Δ^3
0	0	0	1	0	1
1	1	1	1	4	
2	2	2	13		
3	4	28			

a) Find the coefficients b_0, b_1, b_2, and b_3 in the Newton Divided Difference Interpolating polynomial $f_3(x)$. Use the table above and fill in the finite differences.

b) Estimate the function at x=3.

c) An additional data point from f(x) is x = 3, f(3) = 7. Find the true error at x=3.

a)

$$f[x_1,x_0] = \frac{f(x_1)-f(x_0)}{x_1-x_0} = \frac{1-0}{1-0} = 1$$

$$f[x_2,x_1,x_0] = \frac{f[x_2,x_1]-f[x_1,x_0]}{x_2-x_0} = \frac{1-1}{2-0} = 0$$

$$f[x_3,x_2,x_1] = \frac{f[x_3,x_2]-f[x_2,x_1]}{x_3-x_1} = \frac{13-1}{4-1} = 4$$

$$f[x_3,x_2,x_1,x_0] = \frac{f[x_3,x_2,x_1]-f[x_2,x_1,x_0]}{x_3-x_0} = \frac{4-0}{4-0} = 1$$

b)

$$f_3(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1) + b_3(x-x_0)(x-x_1)(x-x_2)$$

$$= 0 + 1(x) + 0(x)(x-1) + 1(x)(x-1)(x-2)$$

$$= x + x(x-1)(x-2)$$

$$f_3(3) = 3 + 3(3-1)(3-2) = 9$$

c) $E_T = f(3) - f_3(3) = 7-9 = -2$

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Problem 3 (40 pts)

A) Estimate the weight of a 6 ft tall high school student using a least squares regression line thru the following data points.

Height (inches) H	Weight (ft) W
72	190
70	175
69	165
74	215
66	145
71	185
68	160

Hint: Use the following table to find the coefficients in the Normal Equations.

H_i inches	W_i lbs	H_iW_i	H_i^2	$\hat{W}_i$	$(W_i-\bar{W})^2$	e_i	e_i^2
72	190	13680	5184	193.6	184.18	-3.6	12.96
70	175	12250	4900	176.4	2.04	-1.4	1.96
69	165	11385	4761	162.9	130.61	-2.9	8.41
74	215	15910	5476	210.7	1487.76	4.3	18.49
66	145	9570	4356	142.1	987.76	2.9	8.41
71	185	13135	5041	185.0	73.47	0	0
68	160	10880	4624	159.3	269.90	0.7	0.49
$\Sigma H_i =$ 490	$\Sigma W_i =$ 1235	$\Sigma H_iW_i =$ 86810	$\Sigma H_i^2 =$ 34342		$\Sigma(W_i-W)^2 =$ 3135.72		$\Sigma e_i^2 =$ 50.72

SST SSE

$\bar{W} = \frac{1235}{7} = 176.429$

B) Complete the table and compute the coefficient of determination.

....

Ans. A) $\hat{W}$ (72) = 193.6, B) r^2 = ________

$$na_0 + (\Sigma H)a_1 = \Sigma W$$
$$(\Sigma H)a_0 + (\Sigma H^2)a_1 = \Sigma HW$$

$$7a_0 + 490a_1 = 1235$$
$$490a_0 + 34342a_1 = 86810$$

$$a_0 = \frac{\begin{vmatrix} 1235 & 490 \\ 86810 & 34342 \end{vmatrix}}{\begin{vmatrix} 7 & 490 \\ 490 & 34342 \end{vmatrix}} = \frac{-124530}{294} = -423.571$$

$$a_1 = \frac{\begin{vmatrix} 7 & 1235 \\ 490 & 86810 \end{vmatrix}}{294} = \frac{2520}{294} = 8.571$$

$$\hat{W} = a_0 + a_1 H$$
$$\hat{W}(72) = -423.571 + 8.571(72)$$
$$= 193.541$$

$$r^2 = \frac{SSR}{SST}$$
$$= \frac{SST - SSR}{SST}$$
$$= \frac{3135.72 - 50.72}{3135.72}$$
$$= 0.984$$