Su 95
EGN 3420

EXAM 2 Name __________________

DO ANY 4 PROBLEMS

Problem 1 (25 pts) Grade: Y___, N___

The following data points are to be approximated by a straight line:

x_i	0	1	1	2	3
y_i	0	1	2	3	3

A) Find the equation of the Least Squares regression line thru the data points.

B) Using the line $y = x$ (not the Least Squares line) for interpolation, find the coeficient of determination r^2.

Work Area

Ans. A) a_0 = _______, a_1 = _______, B) r^2 = _______

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Problem 2 (25 pts) Grade: Y___, N___

For the system of equations below:

$$\begin{array}{ccccccc} x & + & y & - & 2z & = & 0 \\ 2x & + & ky & + & z & = & -3 \\ x & - & 2y & + & z & = & 0 \end{array}$$

A) Find the value of k for which the equations do <u>not</u> have a unique solution.

B) Use the Gauss-Jordan Elimination Method, starting with the augmented matrix (A|b) to find the solution when k=0.

Work Area

Ans. A) k= ______, B) x = ______, y = ______, z = ______

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Problem 3 (25 pts) Grade: Y___, N___

Estimate the function $f(x) = e^x$ at $x=1$ by fitting a 3rd order Newton Divided Difference polynomial thru the four points:

$(0, e^0)$, $(0.5, e^{0.5})$, $(1.5, e^{1.5})$ and $(2, e^2)$.

Express the coefficients b_i, $i=0,1,2,3$ to five places after the decimal point and use them to estimate f(1).

....

Work Area

....

Ans. b_0 = ______, b_1 = ______, b_2 = ______, b_3 = ______

Estimated value of f(1) = ________

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Problem 4 (25 pts) Grade: Y___, N___

A Lagrange polynomial of order 3, i.e. $f_3(x)$, is needed to pass thru the points:

(0,0), (1,1), (2,8), and (4,64). What is the value of $f_3(3)$?

....

Work Area

....

Ans. $f_3(3)$ = _________

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Problem 5 (25 pts) Grade: Y___, N___

Use Trapezoidal Integration with n=5 intervals to estimate the definite integral

$$I = \int_0^1 x^3 dx.$$

Let the result be called I_5. Repeat the process using m=10 intervals with I_{10} as the result. Use the results for I_5 and I_{10} to obtain an improved estimate of I, called $I_{5/10}$. Round all intermediate and final calculations to 5 places after the decimal point.

Work Area

i	x_i	$f_i = f(x_i)$
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Ans. I_5 = _________, I_{10} = _________, $I_{5/10}$ = _________

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①

A)

x_i	y_i	$x_i y_i$	x_i^2
0	0	0	0
1	1	1	1
1	2	2	1
2	3	6	4
3	3	9	9
$\Sigma x_i = 7$	$\Sigma y_i = 9$	$\Sigma x_i y_i = 18$	$\Sigma x_i^2 = 15$

Normal Eqs: $5a_0 + 7a_1 = 9$

$7a_0 + 15a_1 = 18$

$$a_0 = \frac{\begin{vmatrix} 9 & 7 \\ 18 & 15 \end{vmatrix}}{\begin{vmatrix} 5 & 7 \\ 7 & 15 \end{vmatrix}} = \frac{9}{26} = 0.34615$$

$$\hat{y} = \frac{9}{26} + \frac{27}{26} \cdot x$$

$$a_1 = \frac{\begin{vmatrix} 5 & 9 \\ 7 & 18 \end{vmatrix}}{26} = \frac{27}{26} = 1.03846$$

B) $\hat{y} = x$

x_i	y_i	$y_i - \bar{y}$	$(y_i - \bar{y})^2$	$\hat{y}_i$	$y_i - \hat{y}_i$	$(y_i - \hat{y}_i)^2$
0	0	-1.8	3.24	0	0	0
1	1	-0.8	0.64	1	0	0
1	2	0.2	0.04	1	1	1
2	3	1.2	1.44	2	1	1
3	3	1.2	1.44	3	0	0

$\bar{y} = \frac{9}{5}$

$\Sigma(y_i - \bar{y})^2 = 6.8$
SST

$\Sigma(y_i - \hat{y}_i)^2 = 2$
SSE

$$r^2 = \frac{SST - SSE}{SST} = \frac{6.8 - 2}{6.8} = 0.7059$$

c) $\hat{Y} = \frac{9}{26} + \frac{27}{26}X = \frac{1}{26}(9 + 27X)$

X_i	Y_i	$\hat{Y}_i$	$Y_i - \hat{Y}_i$	$(Y_i - \hat{Y}_i)^2$
0	0	0.34615	-0.34615	0.11982
1	1	1.38462	-0.38462	0.14793
1	2	1.38462	0.61538	0.37869
2	3	2.42308	0.57692	0.33284
3	3	3.46154	-0.46154	0.21302

$$\underbrace{\Sigma(Y_i - \hat{Y}_i)^2}_{SSE} = 1.19230$$

$$r^2 = \frac{SST - SSE}{SST} = \frac{6.8 - 1.19230}{6.8} = 0.8247$$

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② $X + Y - 2Z = 0$

$2X + KY + Z = -3$ $\quad A\underline{x} = \underline{b}$

A) $X - 2Y + Z = 0$

$$|A| = \begin{vmatrix} 1 & 1 & -2 \\ 2 & K & 1 \\ 1 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & -2 \\ 0 & K-2 & 5 \\ 0 & -3 & 3 \end{vmatrix} = 3(K-2) + 15 = 3K + 9$$

A unique solution will not exist if $|A| = 0$

$\Rightarrow$ $3K + 9 = 0$

$K = -3$

B) For $K = 0$

$$(A \vdots \underline{b}) = \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 2 & 0 & 1 & -3 \\ 1 & -2 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & -2 & 5 & -3 \\ 0 & -3 & 3 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -2 & 5 & -3 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 3 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{array}\right) \sim \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{array}\right) \end{array}$$

Soln $x = -1$

$y = -1$

$z = -1$

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(3)

i	x_i	$f(x_i)$	Δ	Δ^2	Δ^3
0	0	1	1.29744	1.02369	0.48208
1	.5	1.64872	2.83297	1.98785	
2	1.5	4.48169	5.81474		
3	2	7.38906			

$$f_3(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1) + b_3(x-x_0)(x-x_1)(x-x_2)$$

$$f_3(1) = 1 + 1.29744(1-0) + 1.02369(1-0)(1-0.5) + 0.48208(1-0)(1-0.5)(1-1.5)$$

$$= 1 + 1.29744 + 1.02369(0.5) + 0.48208(0.5)(-0.5)$$

$$= 2.68877$$

(4) Since the Lagrange Polynomial $f_3(x)$ posses thru $(0,0)$, $(1,1)$, $(2,8)$ & $(4,64)$ it must reduce to the cubic x^3,

i.e. $f_3(x) = x^3$

$$\Rightarrow f_3(3) = 3^3$$

$$= 27$$

5) $I = \int_0^1 x^3\,dx$

i	x_i	$f_i = f(x_i)$
0	0	0
1	0.1	0.001
2	0.2	0.008
3	0.3	0.027
4	0.4	0.064
5	0.5	0.125
6	0.6	0.216
7	0.7	0.343
8	0.8	0.512
9	0.9	0.729
10	1.0	1.000

$$I_5 = h\left[\frac{f_0+f_5}{2} + f_1+f_2+f_3+f_4\right] \qquad n=5,\ h=\frac{1-0}{5}=0.2$$

$$= 0.2\left[\frac{0+1}{2} + 0.008+0.064+0.216+0.512\right]$$

$$= 0.26$$

$$I_{10} = k\left[\frac{f_0+f_{10}}{2} + f_1+f_2+\dots+f_9\right] \qquad m=10,\ k=\frac{1-0}{10}=0.1$$

$$= 0.1\left[\frac{0+1}{2} + 0.001+0.008+\dots+0.729\right]$$

$$= 0.252$$

$$I_{5/10} = I_5 + \frac{I_5 - I_{10}}{\left(\frac{k}{h}\right)^2 - 1} = 0.26 + \frac{0.26-0.252}{(1/2)^2-1}$$

$$= 0.24933$$