Su 95

EGN 3420 EXAM 1 Name Key

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Problem 1 (35 pts)

A) Find $f_2(x)$, the second order truncated Taylor Series Expansion of the function $f(x)=x\sin x$ about the point x_0. Leave your answer in terms of x and x_0.

B) For $x_0 = \pi/2$ find the true relative error at $x=1.1x_0$.

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Work Area

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① $f(x) = x \sin x$

A) $f_2(x) = f(x_0) + f'(x_0)(x - x_0) + \dfrac{f''(x_0)}{2!}(x - x_0)^2$

$f'(x) = x \cos x + \sin x$

$f''(x) = -x \sin x + \cos x + \cos x = 2\cos x - x \sin x$

$\Rightarrow f_2(x) = x_0 \sin x_0 + (x_0 \cos x_0 + \sin x_0)(x - x_0) + \frac{1}{2}(2\cos x_0 - x_0 \sin x_0)(x - x_0)^2$

B) $x_0 = \pi/2$ & $x = 1.1 x_0 \Rightarrow x - x_0 = 0.1 x_0 = 0.1(\pi/2)$

$f_2(1.1x_0) = \frac{\pi}{2}\sin\frac{\pi}{2} + \left(\frac{\pi}{2}\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\right)(0.1\,\pi/2) + \frac{1}{2}\left(2\cos\frac{\pi}{2} - \frac{\pi}{2}\sin\frac{\pi}{2}\right)\left(0.1\frac{\pi}{2}\right)^2$

$= \frac{\pi}{2}(1) + (0 + 1)\left(\frac{0.1\pi}{2}\right) + \frac{1}{2}\left(0 - \frac{\pi}{2}\right)\left(\frac{0.01\pi^2}{4}\right)$

$= \frac{\pi}{2} + \frac{0.1\pi}{2} - \frac{\pi}{4}\left(\frac{0.01\pi^2}{4}\right)$

$= 1.1\frac{\pi}{2} - \frac{\pi}{4}\left(0.01\frac{\pi^2}{4}\right)$

$= 0.55\pi - \frac{\pi^3}{1600}$

$f_2\left(1.1\frac{\pi}{2}\right) = 1.7085$

$e_T = \dfrac{f(1.1\pi/2) - f_2(1.1\pi/2)}{f(1.1\pi/2)} = \dfrac{\left(1.1\frac{\pi}{2}\sin 1.1\frac{\pi}{2}\right) - 1.7085}{1.1\frac{\pi}{2}\sin\left(1.1\frac{\pi}{2}\right)}$

$= \dfrac{1.7066 - 1.7085}{1.7066} = -0.0011$

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Problem 2 (35 pts)

A cantilevered beam extending from its clamped end (x=0) to its free end (x=L) has a maximum deflection δ_{max} at the end of the beam, i.e. at x=L. The deflection δ at location $x=\alpha L$ $(0<\alpha<1)$ is related to δ_{max} by the equation

$$f(\alpha)=\alpha^4-4\alpha^3+6\alpha^2-3\delta/\delta_{max}=0$$

Use the Bisection Method to solve for the value of α at which δ/δ_{max} is equal to 0.75. Start with an initial bracket of α_L = 0.5 and α_U = 1.0. Complete 3 iterations and fill in the table below. Round all answers to 5 places after the decimal point.

i	α_L	α_U	α_R	$\lvert e_A \rvert$
0	0.50000	1.00000	0.75	-----
1	0.75	1.0	0.875	0.14286
2	0.75	0.875	0.8125	0.07692
3	0.75	0.8125	0.78125	0.04

(2) $f(\alpha) = \alpha^4 - 4\alpha^3 + 6\alpha^2 - 3\dfrac{\delta}{\delta_{max}} = 0$

$\alpha_L = 0.5$, $\alpha_U = 1$

$\alpha_R = \dfrac{\alpha_L + \alpha_U}{2} = \dfrac{0.5+1}{2} = 0.75$

$\left.\begin{array}{l} f(\alpha_L) = f(0.5) = -1.1875 \\ f(\alpha_R) = f(0.75) = -0.2461 \end{array}\right\} f(\alpha_L)f(\alpha_R) = (-)(-) > 0$

$\Rightarrow \alpha_L = 0.75$

$\alpha_R = \dfrac{\alpha_L + \alpha_U}{2} = \dfrac{0.75+1}{2} = 0.875$

$|e_A| = \left| \dfrac{\alpha_R^{new} - \alpha_R^{old}}{\alpha_R^{new}} \right| = \left| \dfrac{0.875 - 0.75}{0.875} \right| = 0.14286$

$\left.\begin{array}{l} f(\alpha_L) = f(0.75) = -0.2461 \\ f(\alpha_R) = f(0.875) = 0.2502 \end{array}\right\} f(\alpha_L)f(\alpha_R) = (-)(+) < 0$

$\Rightarrow \alpha_U = 0.875$

$\alpha_R = \dfrac{\alpha_L + \alpha_U}{2} = \dfrac{0.75 + 0.875}{2} = 0.8125$

$|e_A| = \left| \dfrac{0.8125 - 0.875}{0.8125} \right| = 0.07692$

$\left.\begin{array}{l} f(\alpha_L) = f(0.75) = -0.2461 \\ f(\alpha_R) = f(0.8125) = 0.0012 \end{array}\right\} f(\alpha_L)f(\alpha_R) = (-)(+) < 0$

$\Rightarrow \alpha_U = 0.8125$

$\alpha_R = \dfrac{\alpha_L + \alpha_U}{2} = \dfrac{0.75 + 0.8125}{2} = 0.78125$

$|e_A| = \left| \dfrac{0.78125 - 0.8125}{0.78125} \right| = 0.04$

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Problem 3 (30 pts)

Use the Newton-Raphson Method to find the value of x where the linear function $y = x + \frac{1}{2}$ and the trigonometric function $y=\cos(x)$ are equal. Start with an initial guess of $x_0 = 0$. Fill in the table below and round the answers for x_i and x_{i+1} to 4 places after the decimal point. Use the rounded answers for x_i to calculate x_{i+1}. Stop when there is no change in x_i (rounded to 4 places after the decimal point).

i	x_i	$f(x_i)$	$f'(x_i)$	x_{i+1}
0	0	-0.5	1	0.5
1	0.5	0.1224	1.4794	0.4173
2	0.4173	0.0031	1.4053	0.4151
3	0.4151	0.0000	1.4033	0.4151
4				
5				
6				

(3) $Y = x + \frac{1}{2}$, $Y = \cos x$

$\Rightarrow x + \frac{1}{2} = \cos x$

$\Rightarrow f(x) = x - \cos x + 0.5$

$f'(x) = 1 + \sin x$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$x_0 = 0$

$$i=0, \quad x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0 - \left[\frac{0 - 1 + 0.5}{1 + 0}\right] = 0.5$$

$$i=1, \quad x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.5 - \left[\frac{0.5 - \cos(0.5) + 0.5}{1 + \sin(0.5)}\right] = 0.4173$$

$$i=2, \quad x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.4173 - \left[\frac{0.4173 - \cos(0.4173) + 0.5}{1 + \sin(0.4173)}\right] = 0.4151$$

$$i=3, \quad x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 0.4151 - \left[\frac{0.4151 - \cos(0.4151) + 0.5}{1 + \sin(0.4151)}\right] = 0.4151$$